Adding a Vorticity Source to the Hasegawa-Wakatani Equations

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Abstract

Instabilities in fusion reactor plasma are an area of ongoing research as they represent the primary hurdle in producing an ongoing fusion reaction for fusion energy production. To study one possible source of these instabilities, computational simulations to solve the modified Hasegawa-Wakatani Equations and the evolution of edge plasmas through time are required. This work shows that the addition of a source of vortices leads to edge instabilities using the TOKAM2D turbulence code. This is an important step into developing our understanding of how instabilities form in plasmas and maintaining a fusion reaction.

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Introduction

Tokamak fusion reactors are a type of fusion reactor which includes the largest planned test reactor in France (Tomabechi *et al.* 1991). Such reactors are torus-shaped devices which use electrical and magnetic confinement to contain a high-temperature plasma where nuclear fusion reactions can take place. A problem in Tokamak reactors is the emergence of turbulences; one source of instabilities that leads to turbulence is the edge plasma, plasma near the outer radius of the torus which is susceptible to edge effects. The region where edge plasmas are relevant is known as the Scrape-Off Layer (SOL).

Method

The Hasegawa-Wakatani (HW) equations are the set of 2D differential equations describing the evolution of the potential and density of edge plasmas in the SOL of Tokamak reactors. This research used the modified Hasegawa-Wakatani (mHW) system, where only local deviations from the average are studied, to investigate the effects of adding an extra vorticity term:

$$\frac{\partial \hat{N}}{\partial t} + \kappa \frac{\partial \hat{\phi}}{\partial y} + \{\hat{\phi}, \hat{N}\} - D\nabla^2 \hat{N} = S + C(\hat{\phi} - \hat{N}), \tag{1}$$

$$\frac{\partial \hat{V}}{\partial t} + g \frac{\partial \hat{N}}{\partial y} + \{\hat{\phi}, \hat{V}\} - D\nabla^2 \hat{V} = C(\hat{\phi} - \hat{N}) - \nu(V - V_0). \tag{2}$$

where N is the number density of charged particles, ϕ is the matter potential, vorticity is $V = \nabla^2 \phi$, and $\{x, y\}$ is the standard Poisson bracket. The presence of a $\hat{}$ indicates the use of the local deviation from the average, for example, $\hat{N} = N - \langle N \rangle$, which is not present in the HW system. The parameters g, D, S, and C are all fixed as $g = S = 0$, $D = 10^{-3}$, and $C = 2$, using the computational unit basis. These parameters determine the strength of each term which act as sources or sinks term and are related to the physical properties of the plasma. This project investigated the effect of adding the term $(V - V_0)$ which forces the vorticity of the plasma towards a predetermined value V_0. ν acts as a parameter which controls the strength of this forcing. The mHW system is used because vorticity is a periodic, local phenomenon.

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To simulate the evolution of instabilities in Tokamak edge plasmas, the turbulence code TOKAM2D (Sarazin *et al.* 2003) was used to numerically solves the HW system (Hasegawa *et al.* 1983), with modifications to solve the mHW system made by Dr. R. Varennes. To check the evolution of the plasmas were physically reasonable, animations of the density, potential, and vorticity were generated.

To investigate how the plasma density flux behaves with the addition of this term, the parameter ν was varied. It was found that negative values resulted in immediate numerical crashes. It was also found that values above 0.3 would result in numerical crashes which occurred so rapidly that no usable data could be collected. These crashes are a result of calculating nonphysical magnitudes in the potential or density of the plasma which could not arise in a Tokamak reactor. To obtain the results shown in Section 1, ν in the range $[0.00, 0.10]$ in steps of 0.005 was used. For greater values of ν, it could be concluded that numerical noise was skewing the data enough that simulations with such values must be discounted.

The form of V_0 was required to be periodic because TOKAM2D uses Fourier transformations to solve partial differential equations. Two forms for V_0 were simulated, one with V_0 as a sinusoidal function and one with $V_0 = 10$, with $\kappa = 0$. These forms were chosen as they are not computationally difficult to calculate and allowed for rapid simulations. A third set of simulations was carried out with $V_0 = 10$ and $\kappa = 1$ to compare the results of adding a vorticity source with the results generated without a vorticity source (Heinonen *et al.* 2020), with modifications in the code to match the simulations of previous research. In the case of $\kappa \neq 0$, this physically corresponds to a set of imposed gradients in plasma density which act as sources of instabilities. Time was measured as the number of periodic cycles of plasma in the Tokamak and each simulation was run for a maximum of 4800 periods. To carry out these simulations the HPCC Wildfly Supercomputing Cluster was used.

Due to the aforementioned Fourier transforms in TOKAM2D, care had to be taken in choosing the time steps used. As ν was increased, the speed at which instabilities arose increased, leading to progressively decreasing simulation times before crashes occurred. Preceding a numerical crash, the density flux rapidly increased in a manner which could potentially skew the data and to take this into account, data immediately before these rapid increases were discounted. Data before plasma instabilities were discounted to avoid skewing the results due to some simulations having longer or shorter stable phases.

Since the simulations were over a region within the SOL, the sum of the vertically averaged density flux over the whole region was used as a measure of the instability as this is the net material flux of the plasma in the SOL. This was plotted to determine how much of the data gathered in each simulation was reasonable and how much had to be discounted. The analysis of these plots and the determination of the start of the instability phase and the beginning of numerical errors was done by manual inspection of each plot. As the fluctuations were centred around zero, the average of the root-mean-square (RMS) of the fluctuations was used to study the behaviour of the density flux.

Results

Across all simulations, it was found that the RMS of the density flux increased to a local maximum at $\nu = 0.025$ (for sinusoidal V_0 and constant V_0 with $\kappa = 1$) or $\nu = 0.030$ (for constant V_0 with $\kappa = 0$) before decreasing to a local minimum at $\nu = 0.065$ or $\nu = 0.070$, respectively. Plots of these data are shown in Figure 1.

The general shape of the plots was consistent across all simulations including a linear decrease between the local maximum and minimum. A notable discrepancy was the density flux for $\nu = 0$ and $\nu = 0.005$. In the case shown for constant V_0, the density flux is initially 0. However, in the case for $V_0 = 10$ and $\kappa = 1$, the initial values were comparable to when $\nu = 0.03$.

Analysis and Interpretation

The matching shapes of the graphs for each simulation suggest that a physically relevant effect occurs when the vorticities of edge plasmas are forced towards a certain value. Furthermore, when using comparable parameters, these individual graphs of density flux agree with previous published research (Ghendrih *et al.* 2022) without vorticity sources, as expected. Where there is no density flux, there may be some threshold value below which stabilising elements overcome the destabilising effects of adding a vorticity source.

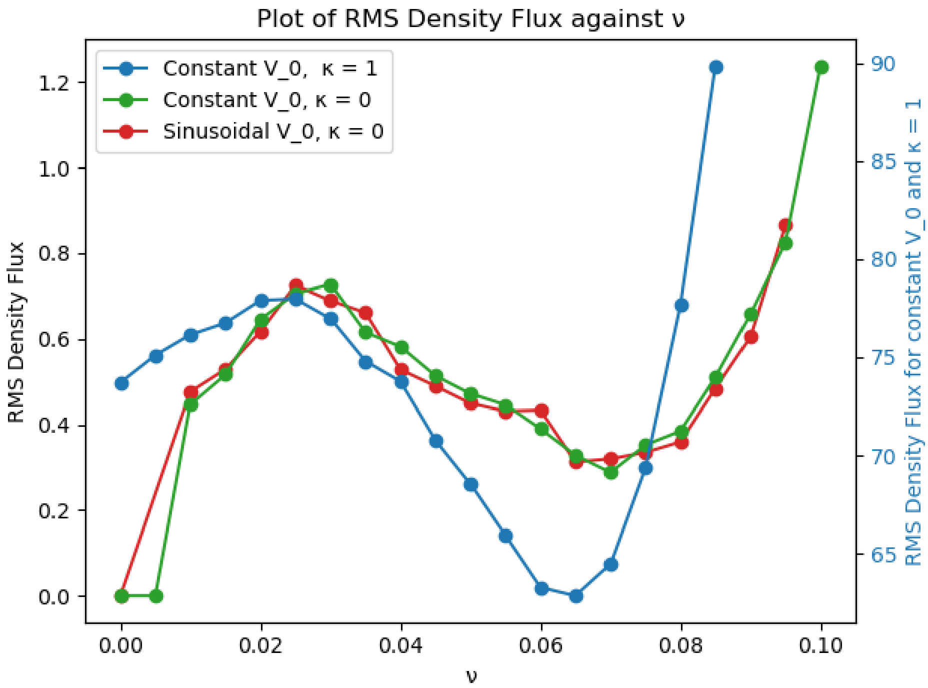

Figure 1: Plot of RMS Density flux against ν with scales for $\kappa = 0$ on the left and $\kappa = 1$ on the right. RMS was calculated from the onset of instabilities and ending before numerical crashes occurred or after 4800 time periods elapsed. All values are given in the computational unit basis.

It is possible to attribute the decrease between the local maximum and minimum to the development of mean velocity shears which have a stabilising effect on the instabilities in the plasma; however, such shears were not visible in the animations generated by the code. Whether this is due to the resolution of the animations being too low or due to a lack of stabilising shears in the simulation is yet to be determined. Following the local minimum, rapid increases in density flux are observed in all simulations. This may be due to the growing effect of numerical errors, though alternatively, it may be due to Kelvin-Helmholtz instabilities arising when mean velocity shears grow too large, studying this behaviour should be the focus of future research.

Conclusion

This work investigated the effects adding a source of vorticity to the modified Hasegawa-Wakatani equations. It was shown that for a variety of vorticity sources, instabilities in the Scrape-Off Layer in Tokamak edge plasmas are produced. This novel result clearly indicates that vortices are a relevant source of instabilities which can cause fusion reactions to fail.

Further research should determine the exact nature of the instabilities formed and whether they can be attributed to mean velocity shear. Work going forward should attempt to extend the number of periods simulated by varying the time step and number of calculations per time step for each value of ν. Additionally, attempts should be made to determine the exact conditions for ν which will result in instabilities by analytically analysing the mHW system. Such work could potentially show us how to maintain an ongoing fusion reaction and unlock the potential that such an achievement possesses.

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References

Ghendrih, P. *et al.* 'Role of Avalanche Transport in Competing Drift Wave and Interchange Turbulence' in Journal of Physics: Conference Series **2397** 1 (IOP Publishing; 2022)

Hasegawa, A. and Wakatani, M. 'Plasma Edge Turbulence' Physical Review Letters **50** 9 (1983)

Heinonen, R. A. and Diamond, P. H. 'Learning How Structures Form in Drift-Wave Turbulence' Plasma Physics and Controlled Fusion **62** 10 (2020)

Sarazin, Y *et al.* 'Theoretical Understanding of Turbulent Transport in the SOL' Journal of Nuclear Materials **313** (2003)

Tomabechi, K. *et al.* 'ITER Conceptual Design' Nuclear Fusion **31** 6 (1991)