Submarine Hydrodynamics For Off-Design Conditions

Serge Toxopeus*†, Maarten Kerkvliet†, Roderik Vogels†, Frans Quadvlieg† and Bart Nienhuis‡

†Maritime Research Institute Netherlands (MARIN)
Wageningen, The Netherlands
e-mail: { s.l.toxopeus | m.kerkvliet | r.vogels | f.quadvlieg } @marin.nl
web page: http://www.marin.nl
*corresponding author

‡Defence Materiel Organisation (DMO)
Utrecht, The Netherlands
e-mail: b.nienhuis@mindef.nl, web page: https://www.defensie.nl/organisatie/dmo

ABSTRACT

Traditionally, submarine hydrodynamic design has focussed mainly on requirements regarding operational range, powering performance and manoeuvring ability for deeply submerged conditions. To improve the effectiveness of the boat, attention is also paid to operating near or at the surface and fortunately, computational tools and experimental methods are available to analyse the performance of submarines at these conditions.

In order to advance submarine hydrodynamics knowledge and tools, DMO and MARIN have conducted a wide variety of bi-lateral or collaborative studies using potential and viscous flow methods and experiments on several submarine hull forms.

In this paper, several examples are presented of the development and use of hydrodynamic tools available during the design and assessment process of future submarines. These examples range from experimental and numerical studies into at-surface and periscope-depth resistance and powering, periscope-depth manoeuvring, high-fidelity flow around the boat during straight flight and manoeuvring motions, wakes of surface-piercing masts, to depth keeping under waves. It is demonstrated how state-of-the-art studies help in advancing the knowledge on submarine hydrodynamics and improving the overall design of modern submarines.

NOMENCLATURE

L	Submarine length [m]
D	Diameter [m]
Fn	Froude number [-]
M'	Non-dimensional pitch moment [-]
X'	Non-dimensional longitudinal force [-]
Z'	Non-dimensional vertical force [-]
λ	Wave length [m]

AMR	Automatic Mesh Refinement
CFD	Computational Fluid Dynamics
CPMC	Computerised Planar Motion Carriage
EFD	Experimental Fluid Dynamics
PIV	Particle Image Velocimetry
RANS	Reynolds-Averaged Navier-Stokes

1 INTRODUCTION

Traditionally, submarine hydrodynamic design has focussed mainly on requirements regarding operational range, powering performance and manoeuvring ability for deeply submerged conditions (van Terwisga and Hooft 1988). However, due to mission requirements and operational profiles, the boat should also perform well in service conditions. To improve the effectiveness of the boat, attention therefore needs also to be paid to for instance operating near or at the surface. Fortunately, computational tools and experimental methods are available to analyse the performance of submarines at these conditions (Renilson 2015). These tools can also be used to better understand the impact of design details on the performance. With the progress of numerical capabilities, both in computational performance as in reliability and accuracy, Computational Fluid Dynamics (CFD, meaning the computation of viscous flows) is more and more becoming the primary design tool of submarines.

In order to advance submarine hydrodynamic knowledge and tools and to support upkeep or successor programmes, DMO and MARIN have conducted a wide variety of bi-lateral or collaborative studies using empirical tools, potential and viscous flow methods and experiments on generic submarine hulls such as the SUBOFF (Toxopeus 2008; Vaz et al. 2010; Toxopeus et al. 2012), Joubert/BB1 (Kerkvliet 2013; Toxopeus et al. 2014; Overpelt and Nienhuis 2014) and BB2 (Overpelt et al. 2015; Carrica et al. 2016; Toxopeus et al. 2019), but also on the existing Walrus class submarines (Bettle et al. 2010). These studies have led to increased applicability and accuracy of the submarine hydrodynamics toolkit.

In this paper, several examples will be shown of the development and use of hydrodynamic tools available during the design and assessment process of future submarines. These examples range from experimental and numerical studies into at-surface and periscope-depth resistance and powering, periscope-depth manoeuvring, high-fidelity flow around the boat during straight flight and manoeuvring motions, wakes of surface-piercing masts, to depth keeping under waves.

The experimental studies comprise captive (for instance CPMC) and free running model tests, while the numerical studies involve empirical tools, potential flow and viscous flow solvers.

The paper will demonstrate how state-of-the-art studies help in advancing the knowledge on submarine hydrodynamics and improve the overall design of modern submarines.

2 EXPERIMENTAL AND NUMERICAL STUDIES INTO AT-SURFACE RESISTANCE AND POWERING

In the early design stage of a submarine, the need exists to determine the required propulsive power and to estimate the operational range of the submarine. These properties can be derived with high fidelity by using model testing or CFD calculations. These do however require detailed knowledge of the submarine's design which is typically not known at this early stage. SubMar is an empirical tool developed by MARIN to predict the resistance and powering requirements of a submarine (van Terwisga and Hooft 1988). By only requiring main dimensions as input, SubMar can be used to explore various design choices in the early design stage.

SubMar is based on empirical relations and is capable of predicting resistance and propulsive power for deeply submerged, periscope depth and surfaced sailing conditions. The latter condition is of importance, as expeditionary diesel-electric type submarines sometimes sail in surfaced condition during (part of) their transit to the area of operations. When sailing in or close to the surface, the wave making resistance is the major component of the total resistance of the submarine.

In recent years, SubMar was improved to better predict the important wave making resistance when sailing at periscope depth and at the surface. Use was made of resistance and powering data at MARIN of eight different submarine designs. The data was obtained using CFD calculations, potential flow calculations, model testing and from full scale trials. Two particularly interesting studies are highlighted in the following paragraphs.

Figure 1. Wave profiles at a speed corresponding to 14 kn on full scale of variants of the BB2 submarine at four L/D-ratios.

2.1 Effect of L/D

From a resistance viewpoint there is an optimum length-to-diameter (L/D) which gives the best compromise between friction and form drag (Renilson 2015). However, due to design and construction requirements, it is often easier for a given submarine displacement to have a relatively longer submarine with a reduced diameter compared to the optimum L/D. As such, it is important to be able to study the impact of L/D on the submarine resistance. The influence of L/D on the submerged resistance has been studied extensively in the past, see Gertler 1950, but the effect on the surfaced resistance also needs to be considered. To improve and validate SubMar for various L/D ratios, model tests have been performed where the L/D ratio of the BB2 submarine was varied from the original 7.3 to 12. Figure 1 shows these four variants being tested at the same speed in MARIN's deep water towing tank.

Based on these model tests where the L/D ratio was varied by increasing the length of the parallel midbody, it is concluded that the resistance increase due to an increased length of the submarine consists mainly of frictional resistance: increasing the length does not increase the wave making resistance. Furthermore, increasing the length increases the wake fraction, while the thrust deduction fraction is not affected. This means that increasing the L/D ratio, without changing the displacement, results in an improved hull efficiency and therefore improved propulsive performance. Hence, a large L/D ratio is preferred for surfaced sailing conditions.

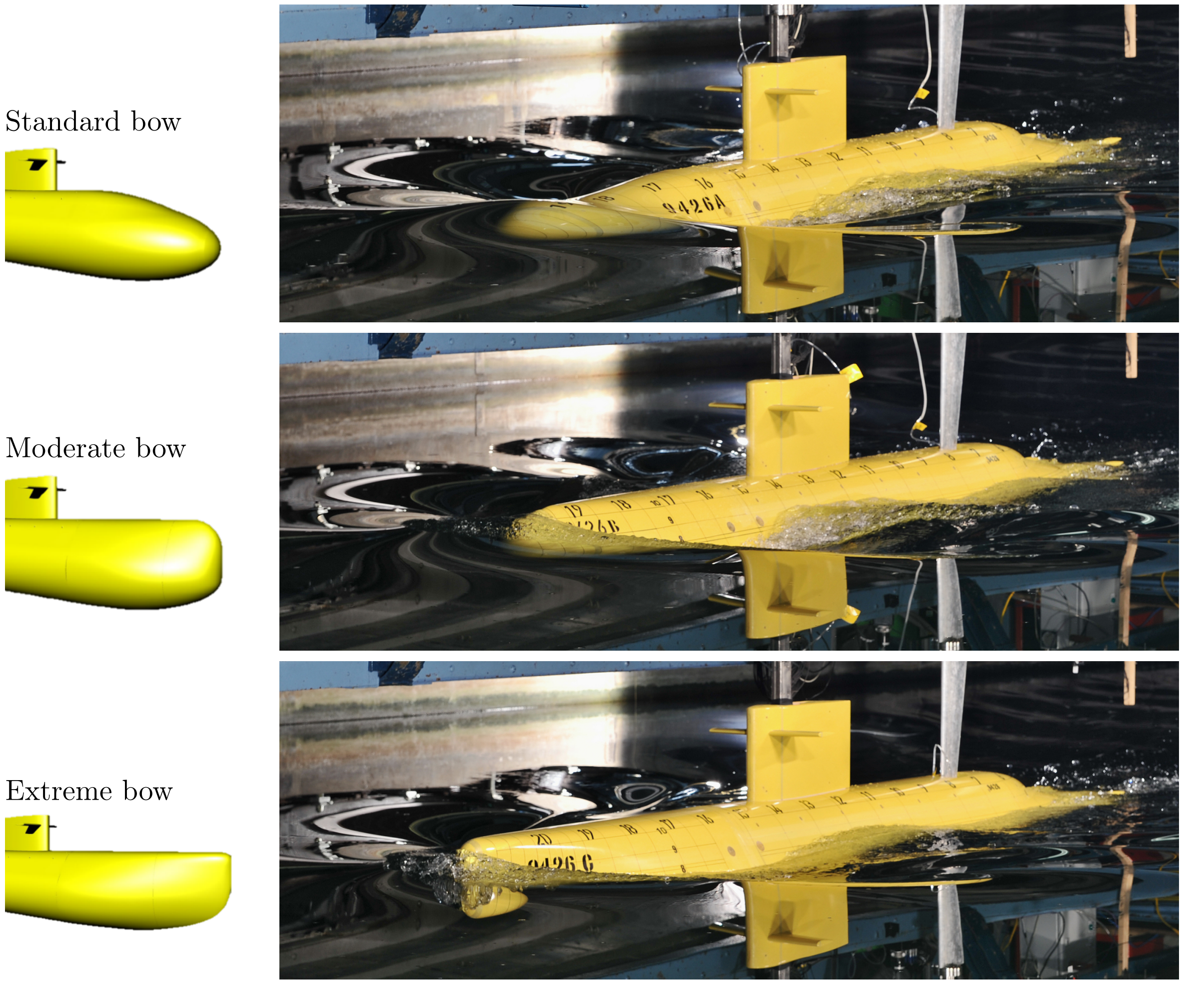

Figure 2. Wave profiles at a speed corresponding to 12 kn on full scale of variants of the BB1 submarine with three different bow shapes of increasing fineness.

2.2 Effect of Bow Shape

Submarine bow shapes are typically optimised for deeply submerged sailing conditions. When sailing at the surface, this bow shape results in a significant wave making resistance. By changing the bow shape it is possible to reduce the bow wave and thus the wave making resistance.

To study the effect of the bow shape on both the surfaced and submerged resistance a series of resistance tests using the BB1 submarine hull form with three different bow shapes has been performed (Overpelt and Nienhuis 2014) and the findings were incorporated in SubMar. Additionally, validation studies of the MARIN in-house CFD code ReFRESCO (Vaz et al. 2009) for surfaced resistance predictions were done. It was shown that the bow shape has a significant effect on the wave making resistance when sailing at the surface. A comparison of the bow wave, which is indicative of the wave making resistance, at the same sailing speed for the tested bow shapes is shown in Figure 2. Overall, it is concluded that the extreme bow shape has a resistance reduction of 44% as compared to the standard bow when sailing at the surface at 12 kn, whilst only showing an increase of 3% in the resistance when sailing deeply submerged. Also previous work by Power 1977 showed that changing the bow shape has relatively little effect on the submerged resistance of bodies of revolution. Of course, careful studies need to be included in order to ensure the performance of sensor and weapon systems placed in or near the bow which can be affected by the changes in bow shape.

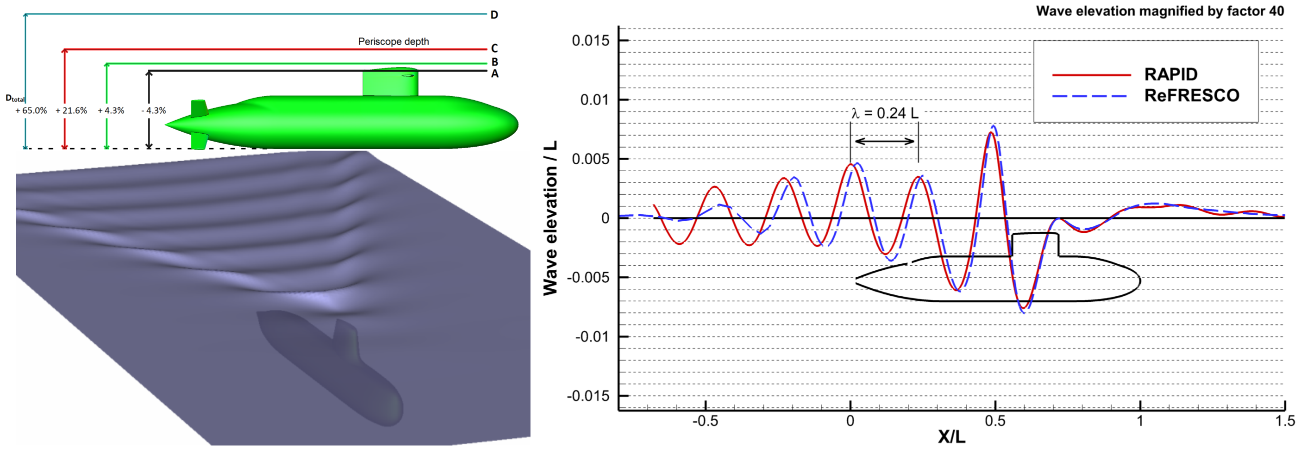

Figure 3. RAPID wave system for a BB2 submarine variant sailing at periscope depth (left) and wave elevation at centreline from RAPID and ReFRESCO computations, $Fn = 0.196$, depth C (right).

3 PERISCOPE-DEPTH OPERATION IN CALM WATER

The analysis of a submarine travelling near the free surface is of interest since many manoeuvres require the submarine to be near the surface to accomplish a given task. Examples of these manoeuvres include snorting, but also ISTAR (Intelligence, Surveillance, Target Acquisition and Reconnaissance) and SOF (Special Operations Forces)operations in hostile waters. Many of these operations are performed while the submarine is in motion, which is why the understanding of the hydrodynamic forces and moments are important to ensure all manoeuvres can be performed safely. As a result, various studies have been conducted at MARIN on submarines travelling near the free surface, both experimental and numerical, see for example Carrica et al. 2016. Another interesting study into exhaust gas plumes during snorting was done by Klapwijk et al. 2017.

One of the aspects of interest at periscope depth is the wave making resistance. Although fully submerged, a submarine sailing at speed at periscope depth will still generate waves and thus experience wave making resistance. This can efficiently be studied using potential flow calculations. An extensive series of calculations has been performed using MARIN's panel code RAPID (Raven 1996). Sailing depth and sailing speed were varied and the obtained wave profiles (an example is given in Figure 3) and wave making resistances for each combination of speed and depth were incorporated in SubMar, such that it is possible to study the performance of a submarine in deeply submerged, periscope depth or surfaced conditions. The waves generated by the submarine can show a favourable interference, which is dependent on the sailing speed. The RAPID results clearly show which speeds are favourable from a resistance point of view, thus giving the operator important information on how to operate the submarine best.

When the tip of the sail is close to the surface, or during manoeuvring conditions, viscous effects become important which cannot reliably be captured by potential flow. Therefore, a need exists for viscous flow computations as well. Viscous flow simulations were performed with ReFRESCO for the fully appended BB2 submarine sailing near the free surface for sailing straight, steady drift and yaw rotation. The four depth conditions are indicated by A, B, C and D, see Figure 3. The main objective was to gain knowledge on the effect of the free surface on the forces and moments acting on the submarine. The results of the ReFRESCO viscous flow computations compared well to RAPID potential flow computations, as shown in Figure 3, and the wave length matches the wave theory for deep water $\lambda/L_{\text{oa}} = 2\pi Fn^2$. The decrease in wave amplitude behind the submarine in the ReFRESCO simulations is due to lack of spatial grid resolution in that wake region.

The non-dimensional resistance and vertical forces (X', Z') and pitch moment (M') are displayed in Figure 4. There is a significant change in the vertical forces acting on the submarine from the conditions C to A. This is due to the interaction of the free surface with the top of the sail and the sailplanes,

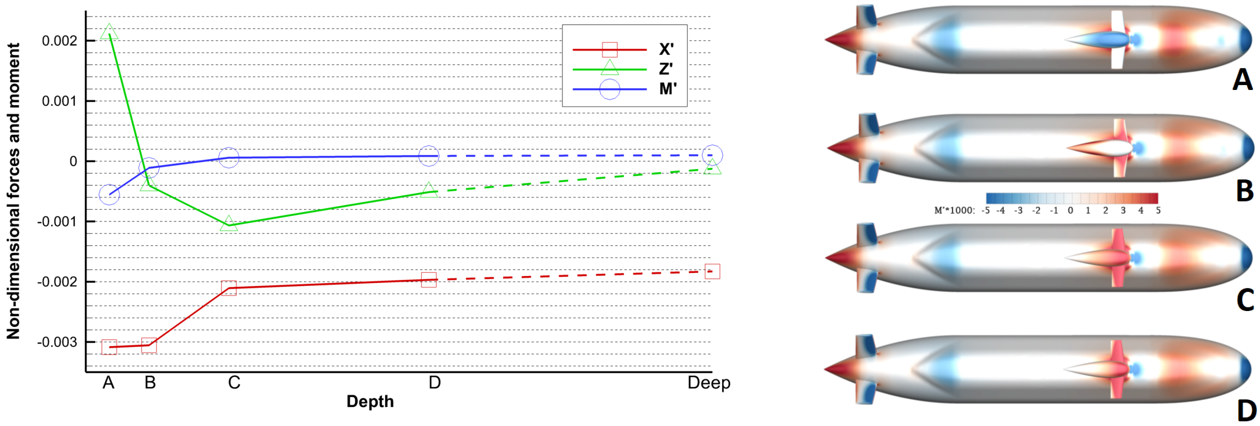

Figure 4. Total forces and moments acting on the submarine at straight sailing conditions (left) and contribution to the pitch moment for each submergence depth (right, red indicates an area which contributes to bow-up and blue vice-versa).

as can be seen in Figure 4. The iso-contours indicate whether an area contributes to a bow-up (red) or bow-down (blue) pitching moment. The out-of-plane force on top of the sail and the lifting forces from the sailplanes are different since a part of the sail is piercing the water surface in condition A and B. As a result the pitch moment changes sign.

More information regarding manoeuvring at periscope depth can be found in Torunski 2018.

4 HIGH-FIDELITY FLOW FIELD PREDICTIONS

CFD can be used to predict the forces and moments acting on a submarine, but also to study in detail the flow around design details. For example, a so-called horse shoe vortex develops at the junction of the casing and sail, due to interaction of boundary layers (Devenport et al. 1992). This horse shoe vortex wraps around the sail and progresses into the propeller plane, possibly causing fluctuations in the loads on the propeller, influencing the propeller performance and noise. CFD provides an opportunity to study the development of the horse shoe vortex and the influence of design changes on the propeller wake. For the BB1 hull form, such a study was conducted by Toxopeus et al. 2014, showing that applying a fillet or cuff in the sail-deck junction area and proper shaping of the sail tip could lead to a reduction of the resistance of several percent and a significant improvement of the propeller wake field. The followed approach can also be used to design fillets for the control planes. Similar studies have been conducted to improve fairings around hull openings or protruding design details such as intercept sonars.

CFD can also be used to better understand features seen in the relation between angle of attack and the forces acting on the fully appended boat. For the BB2, for example, it can be found that at -12° of angle of attack α (which can occur when pitching bow down during diving towards a larger depth), a change of slope in the vertical force and pitch moment curves is present, see Figure 5. This change of slope can lead to unexpected behaviour of the submarine during manoeuvres. Further inspection of the flow field highlights that at this specific angle, the tip vortices coming from the sailplanes interact with the upper rudders and subsequently change the trend in the loads on these rudders, see Figure 6. This phenomenon is especially visible in the pitch moment, due to the large distance of the aft rudders to the centre of reference.

Within the NATO AVT-301 Research Task Group, extensive comparisons between CFD predictions for the BB2 for captive manoeuvres are made (Toxopeus et al. 2019) in order to assess the state-of-the-art of CFD predictions for underwater vehicles. Each member of the group has made predictions for the BB2 at straight flight, 10° drift angle, or steady rotational motion with varying drift angles. The

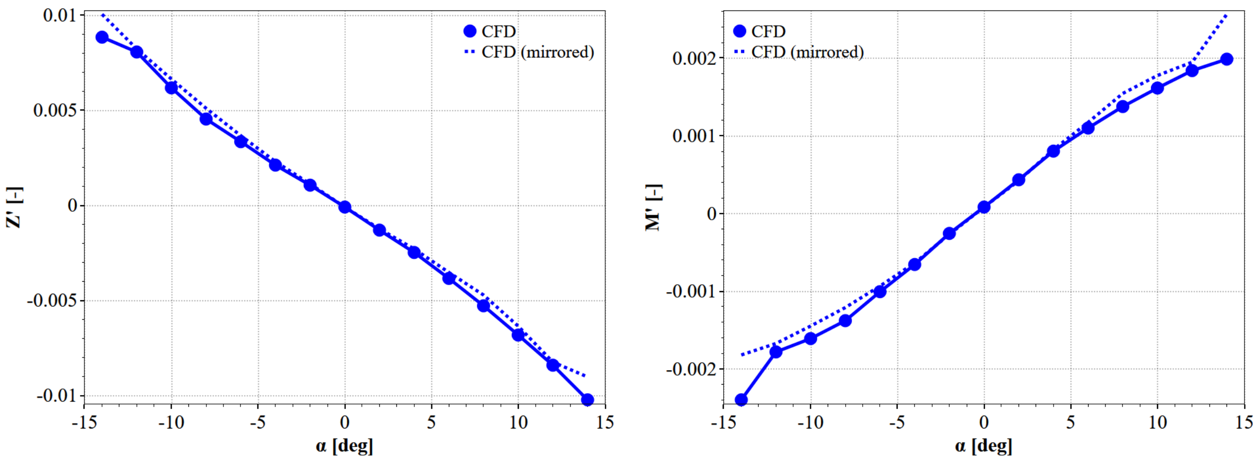

Figure 5. Relation between angle of attack and vertical force (left) and pitch moment (right) on BB2.

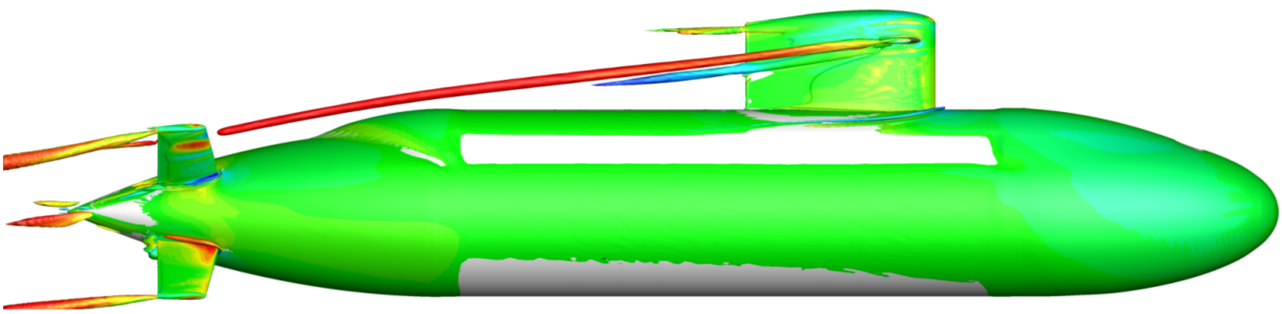

Figure 6. BB2 at $-12°$ angle of attack, visualisation of vortices.

forces and moments acting on the boat and the flow around it have been carefully studied. Although most of the work is still in progress, preliminary conclusions can be made. Regarding the forces and moments on the boat, discretisation errors can be identified when the grid is too coarse. In grids of 14M cells as used in the current study, a discretisation error of 5% was still present. For the flow field itself, and especially near the stern region, it was found that the most efficient results (from computational requirements perspective) with small discretisation errors were obtained with automatic mesh refinement (AMR) or dedicated refinement zones, leading to grids in the order of 100M cells or above. The application of AMR should therefore become standard practice in CFD.

Based on comparisons with wind tunnel tests provided by DSTG (Lee et al. 2018), it is concluded that also the choice of turbulence model is important. For example, changing the turbulence model can result in a change of predicted resistance of up to 10%. Additionally, the turbulence model choice influences the prediction of details in the flow field. The MARIN predictions that showed the best agreement with the experiments for the 10° drift angle were obtained using the RSM SSG/LRR turbulence model (Eisfeld and Brodersen 2005), see Figure 7. More validation of RSM predictions is recommended to assess the performance of this turbulence model.

Preliminary predictions using scale-resolving simulations (for example IDDES or LES) either led to incorrect predictions of the frictional resistance due to relaminarisation of the flow in the computations, or did not lead to improvements compared to conventional RANS predictions.

Recently, DMO has performed measurements in the DNW-NWB aeroacoustic wind tunnel, using high-speed and high-resolution two-dimensional three-component Particle Image Velocimetry (2D-3C PIV) and hotwire anemometry to capture the detailed flow, and pressure probes and microphone arrays to obtain pressure fluctuations and flow and propeller noise. These results will be used in future work for validation of scale-resolving CFD predictions.

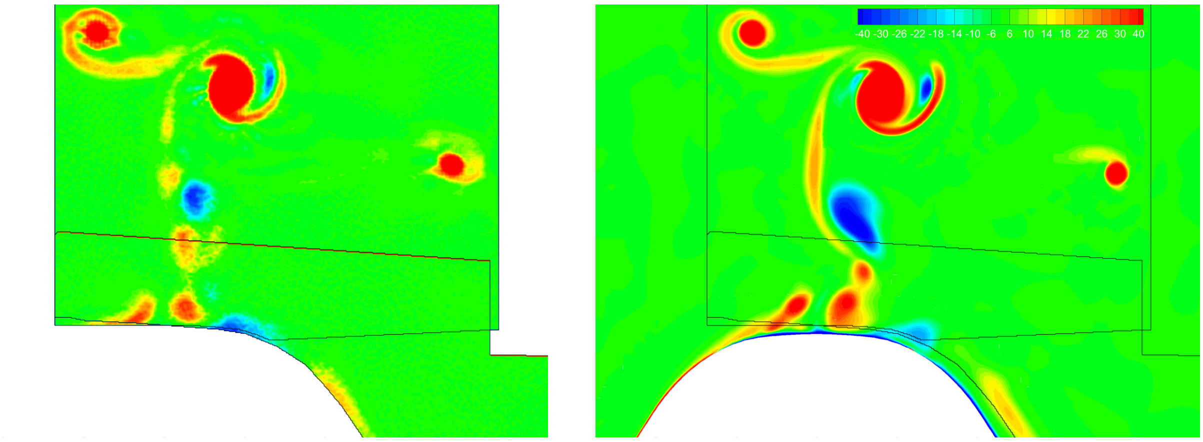

Figure 7. Axial vorticity field behind the sail, DSTG experiment (left) vs ReFRESCO RSM+AMR (right).

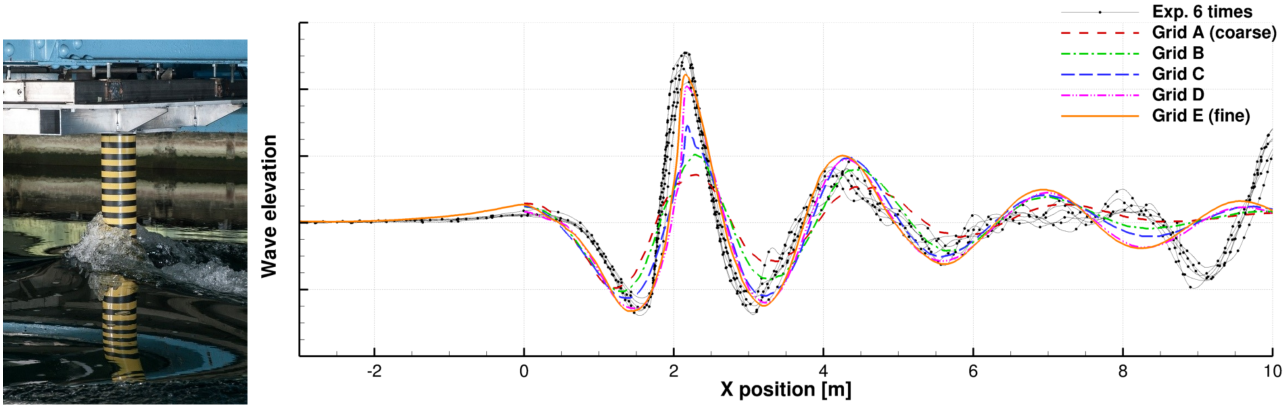

Figure 8. Wave elevation (position relative to the cylinder, positive downstream).

5 WAKES OF SURFACE-PIERCING MASTS

Not only the wave pattern generated by the boat and sail can contribute to increased visibility or resistance during operations at periscope depth, but also masts protruding the water surface, such as a periscope for example. Therefore, an experimental and numerical study into the wave patterns behind a surface piercing mast has been conducted. Inspired by Hay 1947, tests were performed at MARIN at multiple speeds with a schematic surface piercing truncated cylinder with a realistic full scale diameter, clamped at the top to the towing carriage and the tip extending sufficiently far below the water surface to avoid end effects, see Figure 8. During the experiments, the wave elevations behind and next to the mast and the loads acting on the mast were measured, while also the wave pattern was recorded using several video cameras. For selected test conditions, also ReFRESCO computations were carried out.

The experimental results at one of the wave probes along the basin are displayed in Figure 8, together with ReFRESCO results for five geometrically similar grids, showing the sensitivity of local spatial resolution. The elevation of the bow wave at the mast is shown in Figure 9 for RANS $k-\omega$ SST, XLES scale resolved simulations (SRS) (Kok et al. 2004) and XLES with adaptive mesh refinement (AMR) around the mast. The results show that the time averaged wave elevation is comparable for all three cases, so including the RANS, but the instantaneous flow is very different between RANS and SRS. The SRS shows much more dynamic behaviour of the breaking bow wave. Comparing this against the experiments as displayed in Figure 10 gave a good visual match in terms of separation below the water surface and the breaking of the bow wave itself.

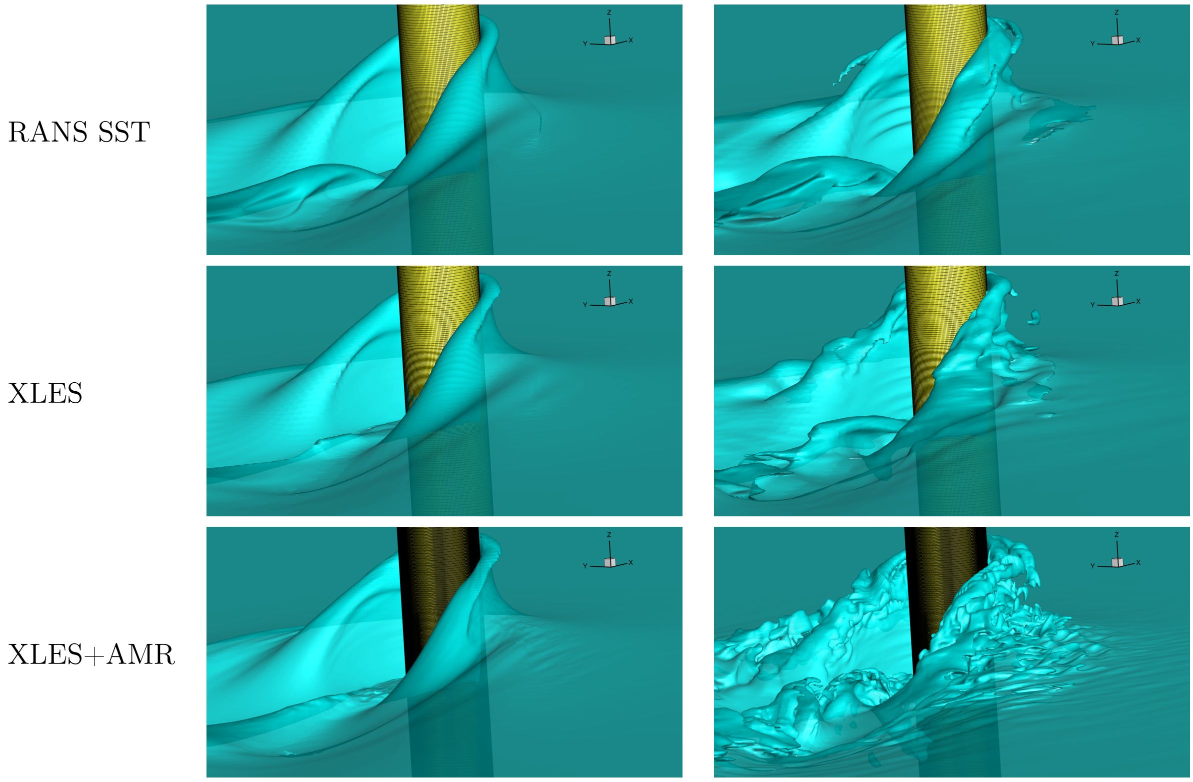

Figure 9. Differences due to turbulence modelling and the use of adaptive mesh refinement (left: time averaged, right: instantaneous).

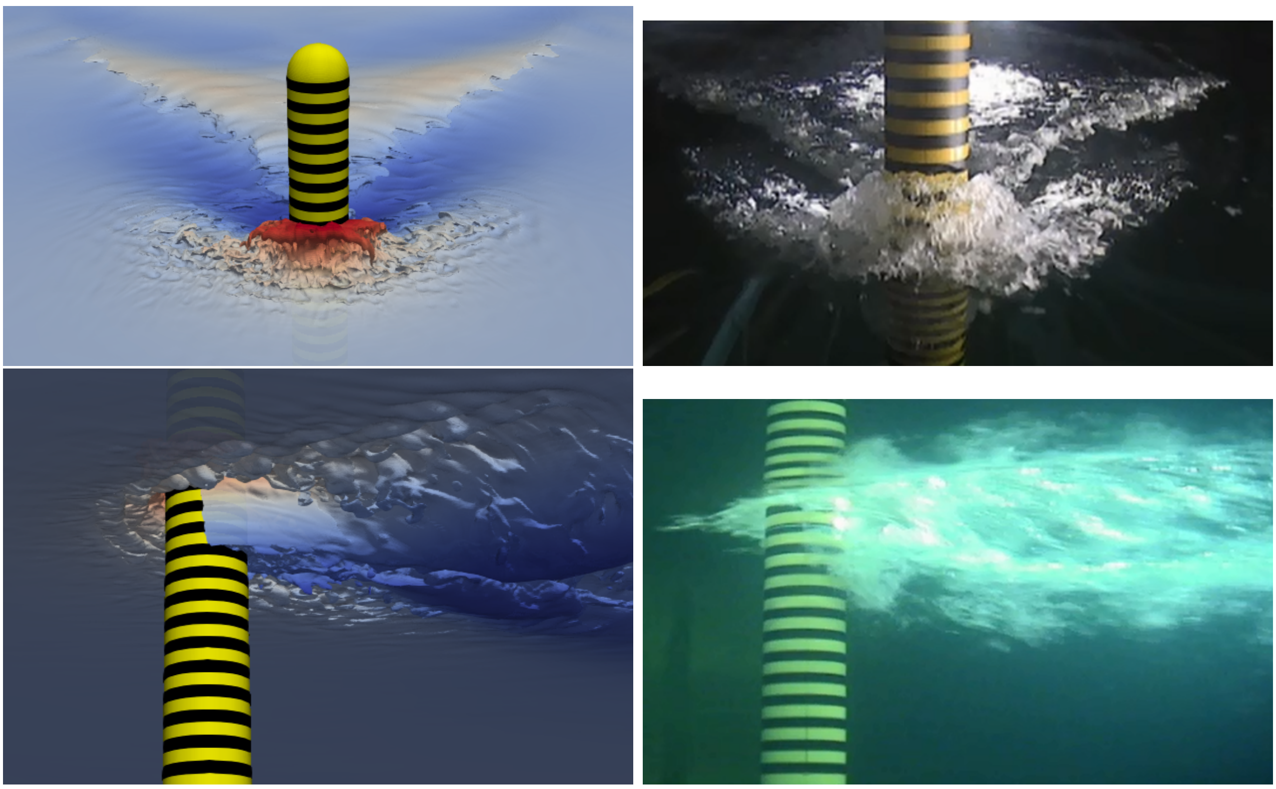

Figure 10. Visual comparison of instantaneous CFD (left) and experimental (right) results.

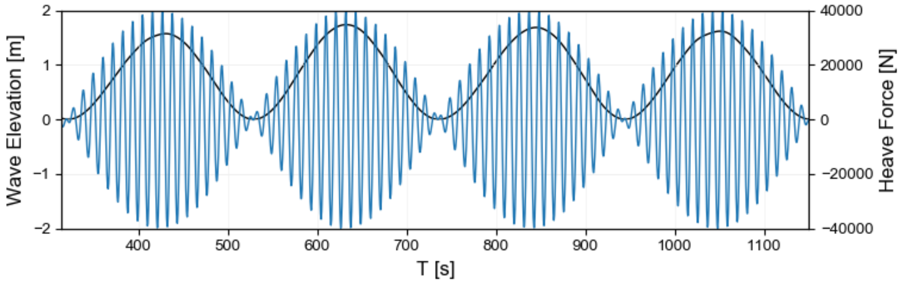

Figure 11. The low frequent heave force acting on a submarine at periscope depth under a bichromatic wave. In blue the wave elevation, in black the low frequent heave force. Note the different scaling of the axes!

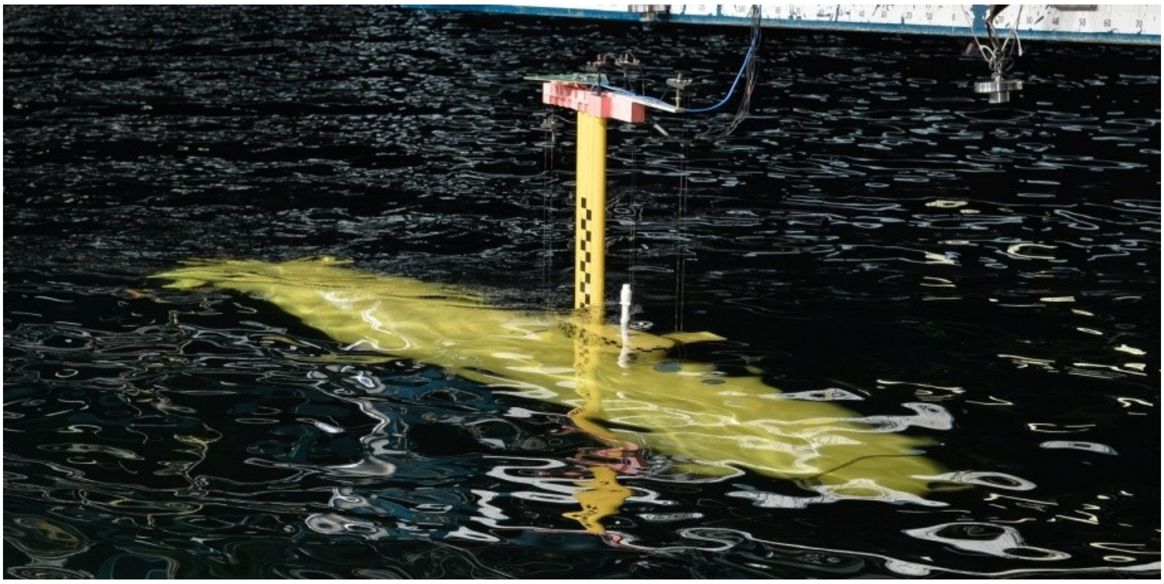

Figure 12. Free running submerged model test with a submarine at periscope depth in irregular bow quartering waves in MARIN's Seakeeping and Manoeuvring Basin.

6 DEPTH KEEPING UNDER WAVES

6.1 Influence of Waves on Depth Keeping Performance

The ability to maintain depth when sailing at periscope depth is of paramount importance to submarines. If the submarine rises too far, the sail will emerge above the water surface which could lead to detection of the submarine. Sinks the submarine too far, periscopes and air intakes will submerge, possibly leading to loss of situational awareness and the forced shutting down of diesel engines. Hence, depth keeping at periscope depth is an important operational capability.

When sailing under waves, a submarine is subjected to wave forces which roughly can be decomposed into forces at the wave frequency and low frequent forces at the frequency of the wave groups. This is illustrated in Figure 11. The low frequent (or 2^{nd} order) forces have a non-zero mean value and tend to pull the submarine to the surface. To accurately model the depth keeping performance of a submarine, it is important to take this 2^{nd} order force into account.

MARIN uses the time domain simulation program SAMSON-XMF (formerly SUBSIM or SAMSON (van Terwisga and Hooft 1988)) to simulate the manoeuvring and seakeeping of submerged submarines. A method has been devised to correctly model the important 2^{nd} order wave drift forces in the time domain simulations. This method is validated against various series of seakeeping model tests at periscope depth with multiple submarine geometries. An example of such a model test with the BB2 submarine is shown in Figure 12.

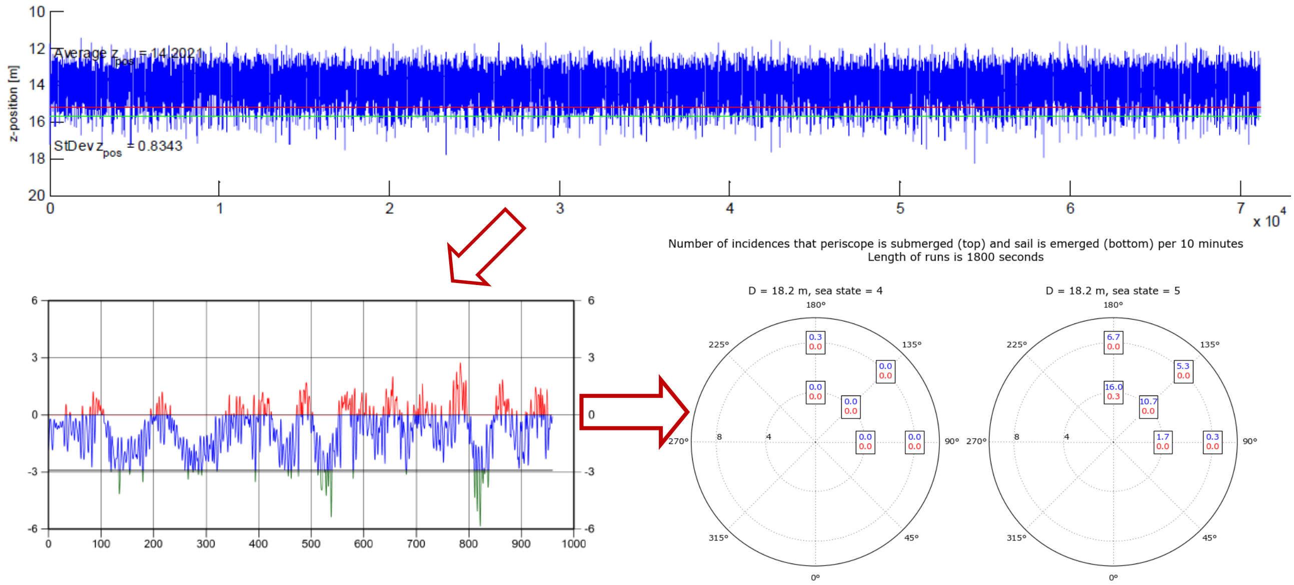

Figure 13. From the heave time trace (top) cases of sail emergence and periscope submergence are automatically extracted (bottom left, sail emergences coloured in green and periscope submergences coloured red). The results from various wave directions and sailing speeds are collected in polar plots (bottom right), showing the results for a sea state. Each small box shows the number of incidents for periscope submergence in blue, and the number of sail emergences in red.

The time domain simulations at periscope depth are automatically processed to identify sail emergences and top of mast submergences. The number of incidences, as well as the total and mean duration are reported as function of sailing speed, sea state and wave direction. An example of this process is shown in Figure 13. The data reduction thus achieved allows for a quantified and easy comparison of the depth keeping performance between various submarine designs. Alternatively, operational limits of the submarine can be defined by comparing the depth keeping performance against operational criteria.

6.2 Effect of Autopilot on Depth Keeping Performance

The depth keeping performance of a submarine is strongly affected by the autopilot used to control the submarine. As the 2^{nd} order wave forces introduce a constant disturbance, either a steady offset to the rudder angles or some type of integrating term in the autopilot is required to counteract this disturbance. The effect of this is seen in Figure 14. As a steady offset requires prior knowledge of the disturbance, which will be dependent on speed, sailing depth and wave direction, the use of an integrating term is preferred.

Changing the autopilot coefficients significantly effects the depth keeping performance. Unsuitable coefficients may lead to unstable behaviour, see for example the purple heave time trace in Figure 15. Optimising the coefficients stabilises the submarines motions whilst minimising rudder usage (as shown by the blue time traces in Figure 15). A fast time simulation model, such as SAMSON-XMF, is invaluable during the autopilot optimisation process. It allows for efficient tuning, where the performance of various coefficients can be quickly compared to find the optimal performance.

7 CONCLUSIONS

With the advance of experimental and numerical tools, the hydrodynamic design of submarines can be improved for the complete operational profile of the boat. Off-design conditions can be taken into account when studying design options, such as improving the performance at the surface without significantly affecting the submerged characteristics of the boat. Especially the fast developments in

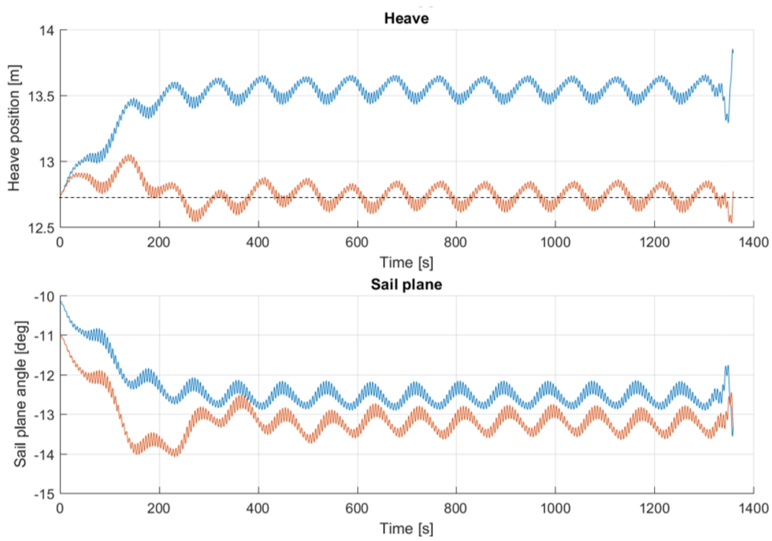

Figure 14. Effect of adding an integrating term to a PID-heave autopilot. With the integrator (in orange), the heave stabilises around the starting value (indicated by the dashed line). Without the integrator (in blue) a steady offset remains.

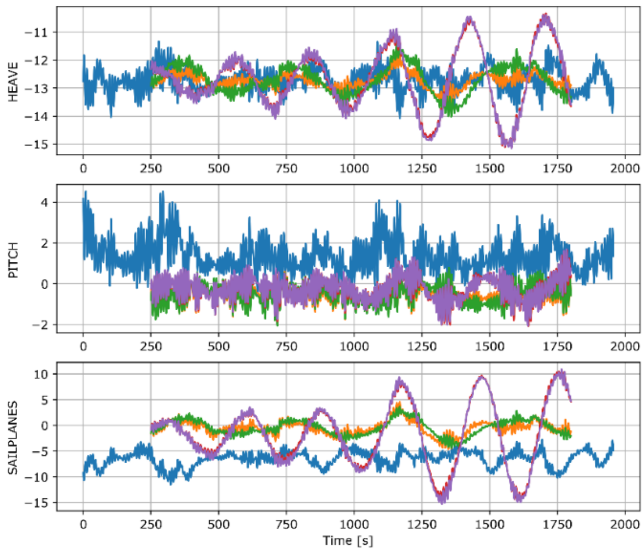

Figure 15. The effect of several autopilot settings on the heave (top), pitch (middle) and sailplane use (bottom) of a submarine sailing in the same wave spectrum at periscope depth, measured in model tests.

CFD accuracy and performance but also advances in model testing techniques allow for detailed studies of the flow around submarines which leads to further understanding of the underlying hydrodynamics, and several examples are given in this paper. With the current hydrodynamic tools, experience and knowledge, upkeep or successor programmes can be better supported than before.

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