IX International Conference on Computational Methods in Marine Engineering
MARINE 2021
I.M. Viola and F. Brennan (Eds)

Prediction of the Vertical Plane Manoeuvring Coefficients for a Submarine when Close to the Surface MARINE 2021

Christopher Polis*[1], Martin Renilson[2] and Dev Ranmuthugala[3]
[1] Australian Maritime College
Email: cpolis@netspace.net.au

[2] Renilson Marine Consulting, Australia
Email: martin@renilson-marine.com

[3] Defence Science and Technology Group, Australia
Email: Dev.Ranmuthugala2@dst.defence.gov.au

Key words: Submarine manoeuvring; close to the free surface.

Summary: The ability to predict the manoeuvring motions of a submerged submarine, particularly in the vertical plane, is very important to ensure safe operation of the submarine. This is usually accomplished using a Coefficient-Based Model (CBM), which allows multiple runs to be conducted in order to determine safe operating boundaries.

The approach using a CBM has been shown to work well for a submerged submarine, however, when close to the surface the presence of the free surface influences the forces on the submarine, predominantly in the vertical plane. Thus, in order to simulate this phenomenon, and understand the safe operation of submarines close to the surface, the values of the vertical plane coefficients as functions of submarine depth need to be known.

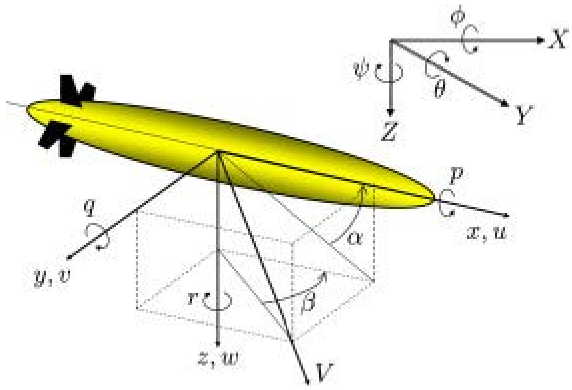
A further complication is that when close to the free surface, the forces on the submarine will be influenced by the trim angle of the submarine. This means that it is not possible to obtain the vertical plane coefficients by simulating the submarine at a pitch angle, as is usually done for the deeply submerged case. In addition, the use of a Planar Motion Mechanism (PMM) to determine the vertical plane coefficients is difficult, as during the sinusoidal motion in the vertical plane, the distance from the free surface is constantly changing.

This paper describes a numerical approach developed to overcome these difficulties, using a concept known as "Fractional Planar Motion" (FPM), where the motion amplitude is limited to a very small fraction of the hull diameter.

First the FPM method is compared to the conventional PMM technique in deep water, to validate this approach. Then the FPM is used to determine the influence of the free surface on the vertical plane coefficients, and the results presented. The proposed approach enables the use of a modified CBM to predict the motions of a submarine when close to the free surface, and hence to efficiently determine safe operating boundaries.

1. NOMENCLATURE

1.1 Directional Conventions



X,Y,Z Earth fixed axes
x,y,z Body fixed axes
$\boldsymbol{u}$, $\dot{\boldsymbol{u}}$ Velocity, acceleration along the x axis
$\boldsymbol{w}$, $\dot{\boldsymbol{w}}$ Velocity, acceleration along the z axis
$\boldsymbol{q}$, $\dot{\boldsymbol{q}}$ Pitch rate, acceleration
θ Angle of Pitch

1.2 Nomenclature

Symbol	Definition	Non-Dimensional Form
A	Amplitude of oscillation	$A^{\star} = \frac{A}{D}; A' = \frac{A}{L}$
D	Diameter of submarine hull	
H	Depth from static free surface to the hull centreline in metres	$H^{\star} = \frac{H}{D}$
K_{z*}	Factor applied to coefficient of axial force as a function of $\boldsymbol{u}^2$	
K_{m*}	Factor applied to coefficient of pitch moment as a function of $\boldsymbol{u}^2$	
K_{mw}	Factor applied to coefficient of pitch moment as a function of $\boldsymbol{w}$	
$K_{m\dot{w}}$	Factor applied to coefficient of pitch moment as a function of $\dot{\boldsymbol{w}}$	
K_{zw}	Factor applied to coefficient of heave force as a function of $\boldsymbol{w}$	
$K_{z\dot{w}}$	Factor applied to coefficient of heave force as a function of $\dot{\boldsymbol{w}}$	
L	Length of submarine hull	
m	Mass of submarine	$m' = \frac{m}{\frac{1}{2}\rho L^3}$
M	Moment about y-axis	$M' = \frac{M}{\frac{1}{2}\rho L^3 U^2}$
$M_{\star}$	Coefficient of pitch moment as a function of $\boldsymbol{u}^{\mathbf{2}}$	$M'_{\star} = \frac{M_{\star}}{\frac{1}{2}\rho L^3}$
M_w	Coefficient of pitch moment as a function of $\boldsymbol{w}$	$M'_w = \frac{M_w}{\frac{1}{2}\rho L^3}$
$M_{\dot{w}}$	Coefficient of pitch moment as a function of $\dot{\boldsymbol{w}}$	$M'_{\dot{w}} = \frac{M_{\dot{w}}}{\frac{1}{2}\rho L^4}$
t	Time	
U	Total velocity	$Fr_L = \frac{U}{\sqrt{gL}}$
x_G	x coordinate of centre of gravity	$x'_G = \frac{x_G}{L}$
Z	Normal force	$Z' = \frac{Z}{\frac{1}{2}\rho L^2 U^2}$

$Z_{\star}$	Coefficient of heave force as a function of $\boldsymbol{u}^2$	$Z'_{\star} = \frac{Z_{\star}}{\frac{1}{2}\rho L^2}$
Z_w	Coefficient of heave force as a function of $\boldsymbol{w}$	$Z'_w = \frac{Z_w}{\frac{1}{2}\rho L^2}$
$Z_{\dot{w}}$	Coefficient of heave force as a function of $\dot{\boldsymbol{w}}$	$Z'_{\dot{w}} = \frac{Z_{\dot{w}}}{\frac{1}{2}\rho L^3}$
ω	Frequency of oscillation	$\omega' = \frac{\omega L}{U}$
ρ	Water density	

1.3 Abbreviations

Abbreviation	Definition
CBM	Coefficient Based Model
CFD	Computational Fluid Mechanics
C-CFD	Captive Computational Fluid Mechanics
FPM	Fractional Planar Motion
PMM	Planar Motion Mechanism

2. INTRODUCTION

The ability to predict the manoeuvring motions of a submerged submarine, particularly in the vertical plane, is very important to ensure safe operation of the submarine. This is usually accomplished using a Coefficient Based Model (CBM), which allows multiple runs to be conducted in order to determine safe operating boundaries. For further details of this see reference [1].

The approach using a CBM has been shown to work well for a fully submerged submarine operating deeply. Traditionally the coefficients have been obtained using captive physical model tests. However, an alternative approach, which is becoming common, is to use Captive CFD (C-CFD) where the CFD simulation is conducted in a similar manner to the physical captive model tests. C-CFD can be used for conventional captive set-ups including: drift angle; rotating arm; and Planar Motion Mechanism tests.

However, when close to the surface the presence of the free surface influences the forces on the submarine, predominantly in the vertical plane. Thus, in order to simulate this phenomenon, and understand the safe operation of submarines close to the surface, the values of the vertical plane coefficients as functions of submarine depth need to be known.

A further complication is that when close to the surface the forces on the submarine will be influenced by the trim angle of the submarine. This means that it is not possible to obtain the vertical plane coefficients by simulating the submarine at a pitch angle, (either using a physical model or C-CFD) as is usually done for the deeply submerged case.

In order to determine the relevant coefficients in the deeply submerged state using a PMM, a pure heave motion could be carried out [1]. This would normally have an amplitude in the order of the diameter of the submarine, and potentially substantially greater. However, in the near surface region, there is a measurable change with depth on these manoeuvring coefficients. Given a motion in the order of the diameter of the submarine induces a change in submergence of that magnitude, this introduces confounding elements into the response being measured. Thus, the use of a conventional PMM to determine the coefficients when close to the surface is not practical.

3. FRACTIONAL PLANAR MOTION

Fractional Planar Motion (FPM) is the numerical modelling of the oscillation of a submarine, wherein the motion is limited in amplitude to a small fraction of the diameter of the submarine. This prescribed motion is specifically intended to address the issues involved in the assessment of coefficients of motion in the near surface. This is not possible using a physical model, however using C-CFD in the computational environment signal noise is substantially reduced, allowing very small forces and moments to be measured distinctly. Position may be controlled to the limits of machine precision; imposed forces are effectively unlimited, with no mechanical flexure; no compromises are required to 'mount' the model; and the results can be obtained without blockage effects. The result is that far smaller amplitude motions can be applied using C-CFD to obtain an outcome than would not be feasible utilising a physical model.

Although this concept is applicable to all oscillating PMM type motions (pure sway, pure yaw, pure heave, pure pitch), this paper will focus upon pure heave only, in line with the general thrust of this work.

3.1 Background

Pure heave motion is generally conducted deeply submerged at a fixed forward velocity with a sinusoidal oscillation in the vertical axis. In this instance, only the case where the submarine *x*-axis is aligned with the global *X*-axis is considered. For small values of $\boldsymbol{w}$, the velocity U is simply the axial velocity $\boldsymbol{u}$**,** and higher order terms in $\boldsymbol{w}$ can be neglected. Thus:

$$\boldsymbol{u}' = \frac{\boldsymbol{u}}{U} = 1; \boldsymbol{w}'(t) = \frac{\boldsymbol{w(t)}}{U} = A'\omega' \cos(\omega' t'); \boldsymbol{v}' = \boldsymbol{q}' = \boldsymbol{r}' = 0 \quad (1)$$

For pure heave motions within the linear region the equations of motion for normal force and pitch moment respectively reduce to:

$$\boldsymbol{Z}'(\boldsymbol{t}) = Z'_{\star}\boldsymbol{u}'^2 + Z'_{w}\boldsymbol{u}'\boldsymbol{w}' + (Z'_{\dot{w}} - m')\dot{\boldsymbol{w}}' \quad (2)$$

$$\boldsymbol{M}'(\boldsymbol{t}) = M'_{\star}\boldsymbol{u}'^2 + M'_{w}\boldsymbol{u}'\boldsymbol{w}' + (M'_{\dot{w}} - m'x'_G)\dot{\boldsymbol{w}}' \quad (3)$$

3.2 Effect of variation of oscillation amplitude and frequency when deeply submerged

In order to consider the use of Fractional Planar Motion for analysis in the near surface region, its use must first be shown in the deeply submerged condition, and the results compared with those using a conventional C-CFD PMM.

A series of numerical simulations [2] was conducted using SUBOFF [3] in the bare hull configuration using C-CFD PMM. Each simulation was conducted at otherwise identical conditions at the same non dimensional oscillation frequency ($\omega' = 0.5$), but $A^{\star}$ was varied from 0.8 to 0.0125 in four geometric steps. Values for the coefficients $Z'_w, M'_w, Z'_{\dot{w}}$ and $M'_{\dot{w}}$ were also obtained using the C-CFD FPM approach. As discussed in [2] the same CFD parameters were used for both approaches, to ensure that any differences were not due to numerical artifacts.

The results from the two approaches are compared in Table 1.

Table 1: Comparison of the Manoeuvring Coefficients using Conventional PMM and FPM

	Z'_w	$Z'_{\dot{w}}$	M'_w	$M'_{\dot{w}}$
PMM	-0.005379	-0.015187	-0.013421	-0.000307
FPM	-0.005435	-0.015162	-0.013421	-0.000303

As can be seen from Table 1, the two methods for predicting the coefficients give very similar results for the deeply submerged condition. Thus, it was considered appropriate to apply the FPM method for the case where the submarine is close to the surface.

3.3 Near surface considerations

In the near surface region, all of the coefficients in equations 2 and 3 above are functions of submergence, $H*$. Thus, equations 2 and 3 are rewritten as equations 4 and 5.

$$\boldsymbol{Z'(t)} = Z'_{\star}(H^{\star})\boldsymbol{u'}^2 + Z'_w(H^{\star})\boldsymbol{u'w'} + (Z'_{\dot{w}}(H^{\star}) - m')\boldsymbol{\dot{w}'} \tag{4}$$

$$\boldsymbol{M'(t)} = M'_{\star}(H^{\star})\,\boldsymbol{u'}^2 + M'_w(H^{\star})\boldsymbol{u'w'} + \left(M'_{\dot{w}}(H^{\star}) - m'x'_G\right)\boldsymbol{\dot{w}'} \tag{5}$$

The result of this is a substantial increase in complexity. For any change in $H^{\star}$, use of these equations requires a prior understanding of how each coefficient is a function of $H^{\star}$. However, if as proposed in FPM, the change in $H^{\star}$ is sufficiently small, then the change in each coefficient over the cycle can be neglected. This then allows determination of each coefficient at different values of $H^{\star}$ to determine the relevant function of the coefficient value with $H^{\star}$.

3.4 Variation of coefficients ($Z'_w, Z'_{\dot{w}}, M'_w,$ and $M'_{\dot{w}},$) as functions of depth

After running a set of simulations similar to the above but near the free surface [2] to confirm that the coefficients also converged as amplitude and frequency approached zero, a simulation was conducted to determine the values of all four coefficients across a series of depth. This, along with the deeply submerged value, allows calculation of the rate of decay towards the deeply submerged value.

A sinusoidal oscillation with an amplitude of 0.0125*D* (as was done for the FPM in the deep case) was superimposed on a gradual progression from a submergence of 1.6*D* to 1.9*D*, with discrete steps of 0.025*D*. At each submergence step, time was allowed to ensure that the forces and moments were quasi-static for that depth. Once a full oscillation had been conducted in that quasi-steady state, a linear transition to the new depth over the period of half a cycle was conducted, and the process repeated for each depth.

The results of this simulation for normal force and pitch moment have been plotted as functions of submergence in Figures 1 to 4, and tabulated in Table 2.

For each coefficient, a steady trend towards the deeply submerged value is observed.

It is therefore proposed that a factor be applied to the deeply submerged values of the coefficients to obtain the force and moment due to motion when close to the surface as shown in equations 6 and 7.

$$\boldsymbol{Z'(t)} = Z'_{\star}K_{z*}(H^{\star})\boldsymbol{u'}^2 + Z'_w K_{zw}(H^{\star})\boldsymbol{u'w'} + (Z'_{\dot{w}}K_{z\dot{w}}(H^{\star}) - m')\boldsymbol{\dot{w}'} \tag{6}$$

$$\boldsymbol{M'(t)} = M'_{\star}K_{m*}(H^{\star})\,\boldsymbol{u'}^2 + M'_w K_{mw}(H^{\star})\boldsymbol{u'w'} + (M'_{\dot{w}}K_{m\dot{w}}(H^{\star}) - m'x'_G)\boldsymbol{\dot{w}'} \tag{7}$$

In equations 6 and 7 the following hydrodynamic coefficients are those applicable to the deeply submerged condition: $Z'_{\star}$; Z'_{w}; $Z'_{\dot{w}}$; $M'_{\star}$; M'_{w}; and $M'_{\dot{w}}$. The factors to be applied to these coefficients to represent the change in the coefficients when close to the presence of the free surface are: $K_{z*}(H^{\star})$; $K_{zw}(H^{\star})$; $K_{z\dot{w}}(H^{\star})$; $K_{m*}(H^{\star})$; $K_{mw}(H^{\star})$; and $K_{m\dot{w}}(H^{\star})$.

The factors to be applied to $Z'_{\star}$ and $M'_{\star}$ ($K_{z*}(H^{\star})$ and $K_{m*}(H^{\star})$) are outside the scope of this work. The values of the other factors, as functions of H^*, are given in equations 8 to 11 for 1.5 < H^* < 3.5. For values of H^* greater than 3.5 the deep water coefficient applies, and all the factors are equal to 1.0.

$$K_{zw}(H^{\star}) = 0.35(H^{*})^2 - 2.34H^{*} + 4.9 \tag{8}$$

$$K_{z\dot{w}}(H^{\star}) = -0.0235(H^{*})^2 + 0.166H^{*} + 0.707 \tag{9}$$

$$K_{mw}(H^{\star}) = -0.024(H^{*})^2 + 0.17H^{*} + 0.7 \tag{10}$$

$$K_{m\dot{w}}(H^{\star}) = -0.326(H^{*})^2 + 2.36H^{*} - 3.26 \tag{11}$$

These values are plotted as functions of H^* in figures 5 to 8.

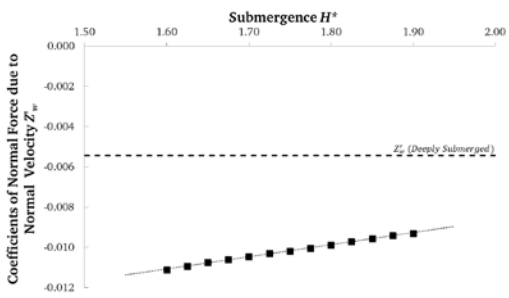


Figure 1: Coefficients of Normal Force due to Normal Velocity as a function of Submergence $\boldsymbol{Fr_L}$=0.512, $\boldsymbol{A^{\star}}$ =0.0125, $\boldsymbol{\omega'}$ = 0.5, SUBOFF in Bare Hull Configuration

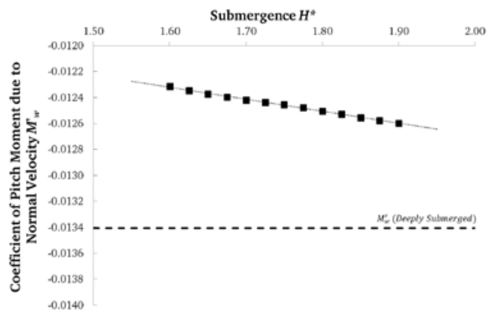


Figure 3: Coefficients of Pitch Moment due to Normal Velocity as a function of Submergence $\boldsymbol{Fr_L}$=0.512, $\boldsymbol{A^{\star}}$ =0.0125, $\boldsymbol{\omega'}$ = 0.5, SUBOFF in Bare Hull Configuration

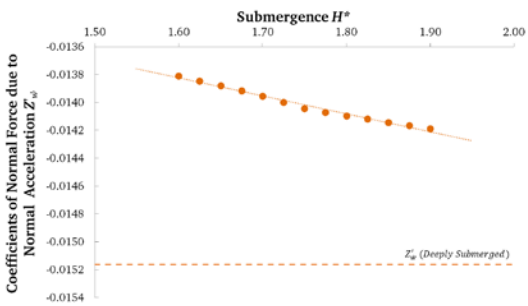


Figure 2: Coefficients of Normal Force due to Normal Acceleration as a function of Submergence $\boldsymbol{Fr_L}$=0.512, $\boldsymbol{A^{\star}}$ =0.0125, $\boldsymbol{\omega'}$ = 0.5, SUBOFF in Bare Hull Configuration

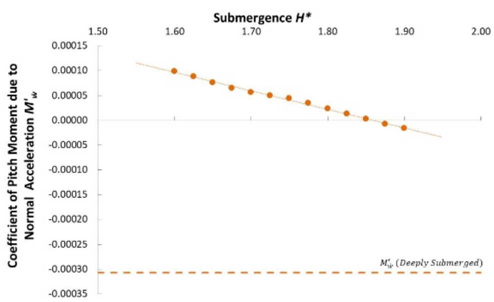


Figure 4: Coefficients of Pitch Moment due to Normal Acceleration as a function of Submergence Fr_L=0.512, $A^{\star}$ =0.0125, ω' = 0.5, SUBOFF in Bare Hull Configuration

Table 2: Coefficients for Bare Hull SUBOFF as a function of Submergence

$H*$	Z'_w	$Z'_{\dot{w}}$	M'_w	$M'_{\dot{w}}$
1.600	-0.011123	-0.013810	-0.012314	0.000099
1.625	-0.010940	-0.013845	-0.012347	0.000088
1.650	-0.010757	-0.013879	-0.012374	0.000076
1.675	-0.010597	-0.013914	-0.012398	0.000065
1.700	-0.010449	-0.013953	-0.012420	0.000056
1.725	-0.010316	-0.013997	-0.012438	0.000049
1.750	-0.010204	-0.014042	-0.012456	0.000044
1.775	-0.010040	-0.014070	-0.012479	0.000034
1.800	-0.009876	-0.014094	-0.012505	0.000024
1.825	-0.009715	-0.014117	-0.012530	0.000013
1.850	-0.009561	-0.014143	-0.012554	0.000002
1.875	-0.009416	-0.014164	-0.012578	-0.000008
1.900	-0.009302	-0.014187	-0.012599	-0.000016
DEEP	-0.005435	-0.015162	-0.013421	-0.000303

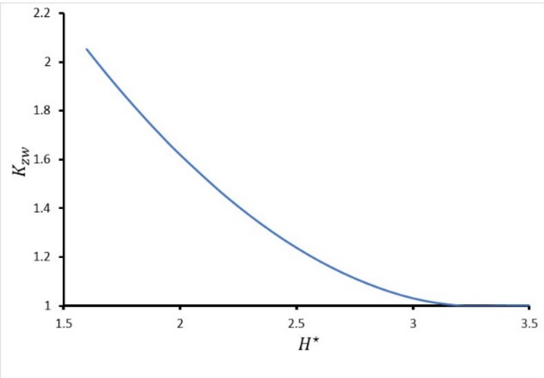


Figure 5: $\boldsymbol{K_{zw}}$ as function of $H*$

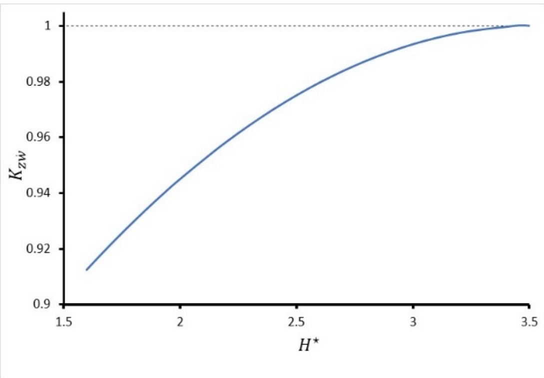


Figure 6: $\boldsymbol{K_{z\dot{w}}}$ as function of $H*$

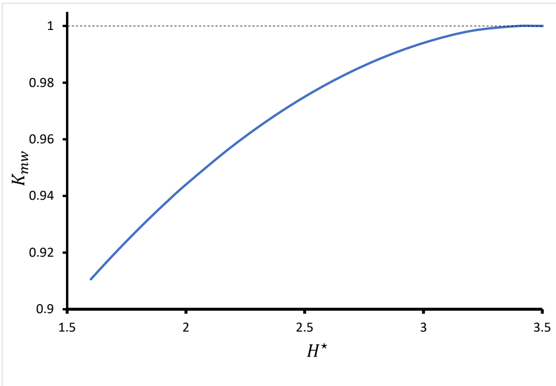


Figure 7: $\boldsymbol{K_{mw}}$ as function of $H*$

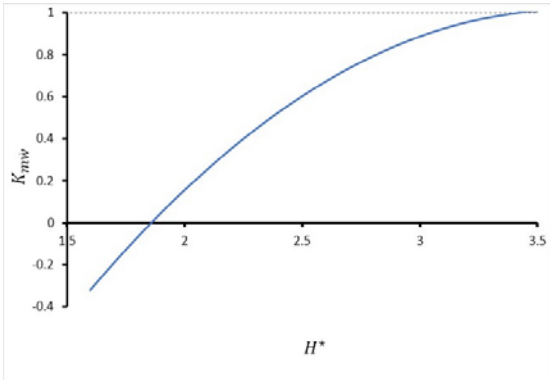


Figure 8: $\boldsymbol{K_{m\dot{w}}}$ as function of $H*$

3.5 Prediction of the heave motions of a submarine when close to the surface

The heave motions of a submarine when close to the surface can be achieved using a CBM based on equations 6 and 7. In these equations the vertical plane hydrodynamic coefficients obtained in the deeply submerged condition: Z'_w; $Z'_{\dot{w}}$; M'_w; and $M'_{\dot{w}}$ are modified using factors which are functions of the non-dimensional distance from the free surface. The values of these factors can be obtained from equations 8 to 11 as functions of the distance from the free surface.

4. CONCLUDING REMARKS

A novel numerical method — Fractional Planar Motion — for the determination of both velocity and acceleration based manoeuvring coefficients in the near surface region has been demonstrated.

Using this method, the change in various manoeuvring coefficients due to operation in the near surface region in for a generic submarine model (the DARPA SUBOFF) has been determined.

The results have been used to develop factors which can be applied to the coefficients in the deeply submerged case to obtain estimates of the values of these coefficients when close to the free surface. These factors are all functions of the distance of the submarine from the free surface. The results are based on a single generic submarine hull shape, and it is recommended that this approach be applied to other hull shapes, to determine whether the factors, as developed here, can be applied to other submarine shapes.

The resulting coefficients can be used in a modified CBM to predict the motions of a submarine in the vertical plane, when operating close to the free surface.

This method should be extended to modelling pure pitching motion, which will enable the evaluation of sufficient coefficients of motion in the near surface region to provide a first approximation of near surface motions.

REFERENCES

[1] Renilson, M.R. *Submarine Hydrodynamics,* (Second Edition) Springer, 2018, ISBN 978-3-319-79056-5.

[2] Polis, C. *Numerical Calculation of Manoeuvring Coefficients for Modelling the Effect of Submarine Motion Near the Surface,* PhD Thesis, Australian Maritime College, An Institute of the University of Tasmania, 2018.

[3] Roddy, R.F. *Investigation of the Stability and Control Characteristics of Several Configurations of the DARPA SUBOFF Model (DTRC Model 5470) From Captive-Model Experiments.* David Taylor Research Center, SHD 1298-08, Maryland, USA, Sept 1990.