

# DATA-DRIVEN IDENTIFICATION OF CRITICAL WAVE GROUPS

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**Abstract.** Accurate and efficient prediction of extreme ship responses continues to be an important and challenging problem in ship hydrodynamics. Probabilistic frameworks in conjunction with computationally efficient numerical hydrodynamic tools such as volume-based and potential flow methods have been developed that allow researchers and ship designers to better understand extreme events. However, the ability of these tools to represent the physics quantitatively during extreme events is limited and not robust to different problems. Therefore, model testing will continue to be important in analysis, and more emphasis will be placed on high fidelity computational fluid dynamics (CFD) simulations. Experiments and CFD both come at well documented *costs* and require a systematic approach to target extreme events. The critical wave groups method (CWG) has been implemented with CFD, and the integration of high fidelity simulations with extreme event probabilistic methods has been previously showcased. The implementation of CWG with CFD is achieved by embedding deterministic wave groups into previously run irregular wave trains such that the motion state of the ship at the moment of encountering the wave group is known. Embedding the deterministic wave groups into an irregular wave train results in a composite wave train that can be evaluated with numerical hydrodynamic simulation tools such as CFD, or even a model test. Though the CWG method does allow for less simulation time than a Monte Carlo type approach, the large number of runs required may still be cost-prohibitive. The objective of the present work is to develop an approach where a limited set of expensive simulations or experiments build a time-accurate long short-term memory (LSTM) neural network model that rapidly identifies critical wave groups that lead to a response exceeding a specified threshold. This paper compares the LSTM modeling approach of building a single neural network for all wave groups to establishing an ensemble of neural networks, each responsible for wave groups with specific parameters. The ensemble approach showcases better accuracy, a higher convergence with respect to data quantity, and produces responses that are representative of the CFD simulations.

## 1 INTRODUCTION

The characterization of extreme ship responses relies heavily on the ability to resolve the hydrodynamics of the wave-body interaction accurately as well as predict the probability of their occurrence. State-of-the-art computational fluid dynamics (CFD) tools have been shown to simulate several ship hydrodynamics problems related to extremes effectively, such as the work with the Design Loads Generator (DLG) in [1] for large roll and capsizing and broaching simulations in [2]. Analysis with CFD strives to provide a more quantitative depiction of extreme events without the empiricism and modeling inherently required by more efficient linear and nonlinear potential flow methods. However, with higher fidelity comes larger computational cost. The computational cost renders CFD impossible for Monte Carlo type approaches for analyzing extremes. Even within probabilistic frameworks like the critical wave groups (CWG) method leveraged to predict the probability of extremes, utilizing CFD can still be impractical due to the high computational cost.

Previous work has addressed the computational cost of CFD by building surrogate models of the dynamical process within a probabilistic framework. [3, 4] utilize Gaussian process regression (GPR) within a sequential sampling framework to map the wave group parameter space to a maximum dynamical response. The surrogate model then predicts the extreme statistics. The present work investigates the ability of a long short-term memory (LSTM) neural network to act as a surrogate in the framework developed in [5] that implements the CWG method with CFD. LSTM neural networks have been shown by [6, 7] to reproduce the dynamical response time histories of vessel motions due to wave excitations successfully. However, previous utilization of LSTM neural networks in [6, 7] only consider random irregular waves and the resulting motions are not extreme. Therefore, employing LSTM neural networks for predicting extremes is the focus of this paper.

The objectives of this paper are to evaluate the ability of an LSTM neural network to predict extreme motions within the CWG-CFD probabilistic framework developed in [5], to investigate different neural network modeling approaches, and to understand the effects of training data quantity on the accuracy of the neural network. The remainder of the paper is organized as follows. A brief summary of CWG and the CWG-CFD framework from [5] is presented, followed by an overview of the considered neural network architecture and methodology. Then, comparisons of different modeling approaches and data quantities are detailed to evaluate the ability of an LSTM neural network to act as a surrogate for CFD simulations of the extreme roll of a midship section of the Office of Naval Research Tumblehome (ONRT) hull form.

## 2 CWG-CFD FRAMEWORK

### 2.1 Critical Wave Groups Method

The CWG method originates with [8] for regular waves and was recently implemented with irregular waves in [9]. The key premise of the methodology asserts that the probability of a response exceeding a threshold is equal to the probability of all of the wave groups and states of the ship at the moment of wave group encounter that cause a threshold exceedance. The CWG method formulates this probability by identifying wave group and encounter condition pairs that result in a near-exceedance of the specified threshold. [9] contains a detailed description and

derivation of the probability calculations for the CWG method in the present paper.

An important aspect of the CWG method is the construction of deterministic wave groups. The waves within the group are treated as Markov chains, where each wave is defined by its height  $H$  and period  $T$ . The largest wave of the group is selected  $(H_c, T_c)$ , and the successive waves are determined based on the statistics for the given wave condition of the most likely next wave. This process can be repeated for all successive waves to build an entire group of  $j$  waves. This procedure allows entire wave groups to be constructed solely from the height and period of the largest wave in the group as well as the number of waves preceding and following the largest wave. However, the present Markov chain construction only produces predictions of the height and periods of the waves in the group and does not contain any information about the shape. [10] presented the methodology shown in Fig. 1, where the crest and trough of each wave are assumed to each be half of the wave height, the time derivative of wave elevation at the crest and trough is zero to ensure they are peaks, the crest and trough occur at the center of the interval defined by the successive zero-crossings, and the zero-crossings occur at instances of half of the current wave period. With the Markov chain and geometric constraints, the entire waveform is interpolated. A Fourier basis is applied in this paper, like the work of [9], rather than the Karhunen-Loève waveform approach in [10] and annotated in Fig. 1. The differences between the two interpolation methods are negligible in the present work.

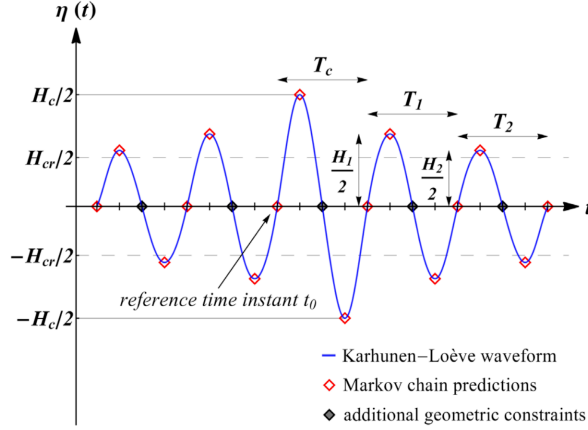


Figure 1: Construction of wave groups with Markov chains [10].

Fig. 2 shows an example of a series of wave groups constructed utilizing the methodology outlined in Fig. 1 with a period of  $T_c = 15$  s for the largest wave and a wave group run length of  $j = 3$ . By only varying the height ( $H_c$ ) of the largest wave in the group, the Markov chain methodology describes the successive waves. Thus, the wave groups are characterized by prescribing  $H_c$ ,  $T_c$ , and  $j$  and interpolating between the peaks and troughs with a Fourier basis.

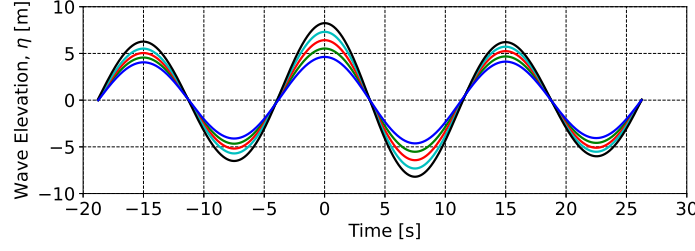


Figure 2: Ensemble of different wave groups with variation of only the largest wave height.

## 2.2 Framework

The framework outlined in Fig. 3 was first developed in [5] to implement the CWG method with CFD. The framework begins with prescribing a seaway of interest and producing random irregular waves from that particular seaway. The irregular waves are interrogated to determine the statistics of successive wave heights and periods to form the transition kernels for the Markov chain wave group construction. Also, CFD simulations of ship motion are performed with the random irregular waves to calculate the probability of the encounter conditions.

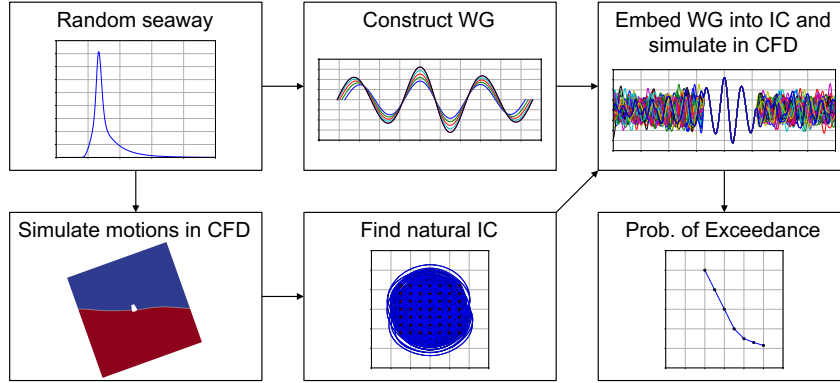


Figure 3: Flow chart of CWG-CFD process.

The response time histories of the CFD simulations then identify occurrences of prescribed encounter conditions and the wave train that caused them. With the exception of the work in [5], previous implementations of CWG used single degree-of-freedom ordinary differential equations (ODE) to model roll. For an ODE-based dynamical equation, the state of encounter can be specified as an initial condition and the wave group can be instantiated impulsively as an input. For a model test or CFD implementation of CWG, the encounter condition and wave group must be physically realizable and therefore, cannot start impulsively. Thus, [5] introduced the concept of the natural initial condition, where simulations of the ship in random waves are scanned for different encounter conditions. Then, the waves leading to the encounter conditions are identified and the different deterministic wave groups are embedded into the irregular wave train, such that the encounter condition occurs at the moment the ship encounters the wave group. Fig. 4

demonstrates an ensemble of different composite wave trains with the same embedded wave group. Each portion of the composite wave train that corresponds to the natural initial condition will result in a different selected encounter condition at the moment the ship encounters the wave group.

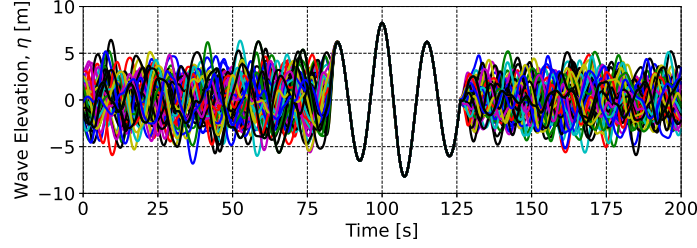


Figure 4: Ensemble of different waves with corresponding different natural initial conditions for the same wave group.

After embedding the wave groups into the natural initial condition wave trains, the resultant composite wave trains are simulated in CFD to determine the corresponding ship response, identify the critical wave group (that which leads to a threshold exceedance), and to calculate the probability of exceedance in accordance with the formulation developed in [9]. For each  $T_c$ ,  $j$ , and encounter condition, a unique value of  $H_c$  describes the critical wave group where a threshold exceedance occurs. Fig. 5 showcases how a critical wave group is identified for a given response  $\phi$  exceeding a threshold  $\phi_{crit}$ . Each curve represents a given wave group elevation and the corresponding response for a series of composite wave trains that only vary in  $H_c$ . For a given threshold, a unique wave group (shown in red) denotes the critical wave group where any larger group will also lead to an exceedance. The procedure in Fig. 5 is repeated for every  $T_c$ ,  $j$ , and encounter condition at each desired response threshold. The identification of the critical wave group is the most important aspect of the CWG method and requires an accurate depiction of the ship hydrodynamic response to the prescribed composite wave excitation. Any surrogate model of the CFD simulations, such as an LSTM neural network, must maintain a sufficient degree of accuracy to identify the phenomena in Fig. 5.

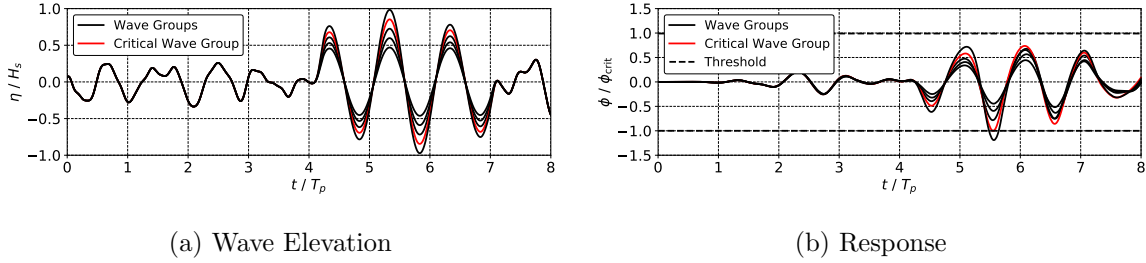


Figure 5: Identification of a critical wave group for a given set of composite wave trains utilizing the same encounter conditions and wave groups with the same parameters  $T_c$  and  $j$ .

### 3 NEURAL NETWORK ARCHITECTURE

The current paper builds off the work of [7] and utilizes an LSTM neural network, trained with CFD simulations, to predict ship responses. LSTM and Recurrent Neural Networks (RNN) in general have been applied to different complicated datasets for system identification. In particular, LSTM neural networks are capable of learning and predicting time-history data and sequence learning tasks such as natural language processing. LSTM are well suited for the prediction of ship responses due to wave excitation because of their ability to build relationships between the time-sequencing in datasets. Predictions made by LSTM are not only based upon the current state, but are also influenced by data at previous time steps as well. The previous work of [6, 7] has demonstrated the accuracy of the LSTM neural networks for predicting ship motions simulated by CFD. The work of [6, 7] also showcases how expensive CFD simulations can be reduced in dimensionality, to only the dynamic ship motion quantities of interest, and still produce comparable response time-histories. Fig. 6 shows an example neural network architecture from [7] with five LSTM layers followed by a dense layer. The inputs and outputs of the model are labeled as  $x_t$  and  $\hat{y}_t$  respectively, and  $t$  is the time step index that ranges from 1 to  $T$ . The state and output of LSTM cell  $n$  at time index  $t$  are denoted by  $C_t^n$  and  $h_t^n$  respectively. The dense layer in Fig. 6 employs a linear activation function and outputs the final result of the LSTM model.

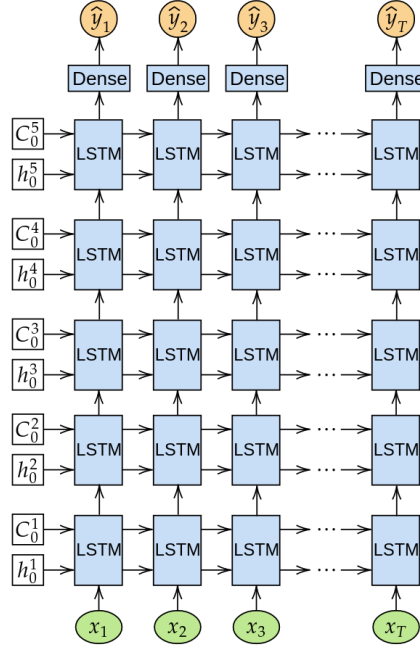


Figure 6: Neural network architecture from [7].

The present paper employs a neural network model architecture similar to Fig. 6 and the methodology of [7], but with two LSTM layers and 50 cells per layer. The architecture is implemented with the toolbox Keras [11]. Prior to building the LSTM models, the inputs and outputs are standardized, such that the mean and standard deviation of each is zero and unity

respectively. The training of the models use an *Adam* optimizer [12] with the mean-squared error averaged loss function shown in Eqn. 1, where  $\hat{y}$  is the prediction of the output sequence and  $y$  is the true value.

$$L(\hat{y}, y) = \frac{1}{T} \sum_{t=1}^T (\hat{y}_t - y_t)^2 \quad (1)$$

The models utilize the wave elevation time-history of different composite wave trains at the inlet of the CFD domain as input into the models, and the output is the heave and roll response time-histories. The resulting trained neural network model is capable of receiving a given composite wave train time-history at the CFD domain inlet and provides predictions of the temporal response of the heave and roll motions. This differs from other work in [6, 7] with LSTM models, as the composite wave trains will inherently lead to extreme motions by design, and only random irregular wave excitations were considered in the previous work. The presented idea also differs from the work of [3, 4] utilizing GPR for extremes, as the LSTM models provide the temporal response and not only the resulting maximum response to each considered wave train.

Two modeling approaches are considered in the current paper. Approach A utilizes one model to learn the input/output relationship between wave elevation and the ship's heave and roll. Input into the models with the A approach will include different combinations of encounter conditions and wave groups. Approach B includes an ensemble of models, where a single model is built for wave groups with the same parameters  $T_c$  and  $j$ . Each individual model only trains with data for wave groups corresponding to its respective  $T_c$  and  $j$ . When repeated for varying values of  $T_c$  and  $j$ , the ensemble of models can predict responses for a range of different wave groups. The two modeling approaches are investigated in the present paper for both accuracy and convergence with respect to the total simulations utilized for the training dataset.

## 4 RESULTS

Different LSTM neural network methodologies are demonstrated for the same case study considered in [5] of a two-dimensional (2-D) midship section of the ONRT geometry [13] that is free to heave and roll. The different LSTM neural network models are evaluated for their ability to represent the dynamical ship response predicted by the CFD simulations for all the composite wave train excitations that correspond to the full probabilistic analysis of the CWG-CFD framework in [5]. The present work utilizes the open-source toolkit OpenFOAM® to simulate the ship motion response due to nonlinear generated seaways with customized CFD solvers and libraries developed by the Computational Ship Hydrodynamics Laboratory (CSHL) at The University of Michigan [14, 15]. The ship section is exposed to a seaway from a JONSWAP spectrum [16] with a peak enhancement factor  $\gamma = 3.3$ , a significant wave height  $H_s = 7.5$  m, and a peak modal period  $T_p = 15$  s corresponding to a Sea State 7 [17].

For each composite wave train, the heave and roll predictions utilizing the A and B modeling approaches are compared against the CFD simulations for training data set sizes of 50, 100, 200, and 400 simulations of randomly selected composite wave fields. The  $L_2$  and  $L_\infty$  errors, described in Equations 2 and 3 respectively, compare the accuracy of the response time-histories predicted for each run, where  $y = (y_1, \dots, y_T)$  is the values from the CFD simulations,  $\hat{y} = (\hat{y}_1, \dots, \hat{y}_T)$  is

the prediction made by the LSTM neural network models,  $i$  is the index for a single time step, and  $T$  is the total number of time steps. The  $L_2$  error measures the accuracy of the models in an average sense, while the  $L_\infty$  error is purely considering the maximum difference between the predictions and the CFD simulations.

$$L_2(y, \hat{y}) = \sqrt{\frac{1}{T} \sum_{i=1}^T (y_i - \hat{y}_i)^2} \quad (2)$$

$$L_\infty(y, \hat{y}) = \max_{i=1, \dots, T} |y_i - \hat{y}_i| \quad (3)$$

Fig. 7 shows the  $L_2$  error comparison for both modeling approaches A and B, trained with 50, 100, 200, and 400 simulations. Each circle represents the response predicted for a single composite wave train. Overall, the magnitude of the  $L_2$  error is less for the B models with an increase in training data volume clearly demonstrating improvements in the predictions. However, the A approach shows little difference between the various training data quantities and results in a larger  $L_2$  error in roll. The small difference between training data quantities indicates that the A approach models are more than likely converged with few runs and the addition of any other data will not result in a more accurate model. Additionally, since each composite wave train contains a deterministic wave group meant to excite extreme motions, the larger  $L_2$  errors are more than likely due to the differences in the peak values of each particular response.

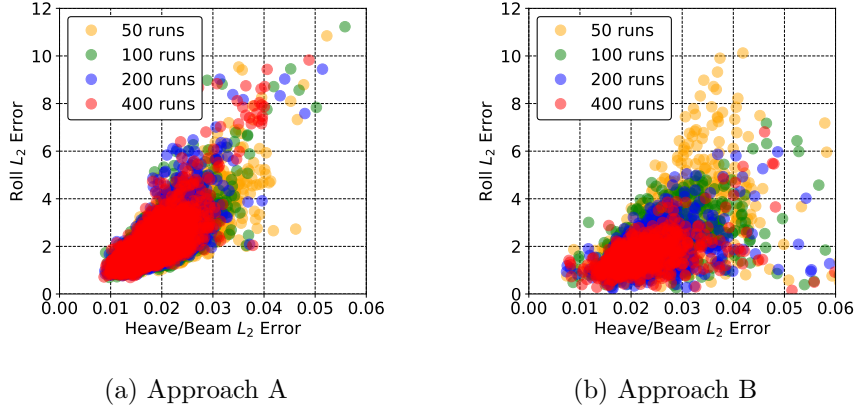


Figure 7: Comparison of the  $L_2$  error for each composite wave run with different LSTM models and varying amounts of training data.

Fig. 8 shows the  $L_\infty$  error comparison for both approaches A and B, trained with the different quantities of data. Similar to the  $L_2$  error, the A approach shows very little difference between varying data quantities, while the B models demonstrate that the addition of data results in a more accurate prediction of the extreme motion response of both roll and heave.



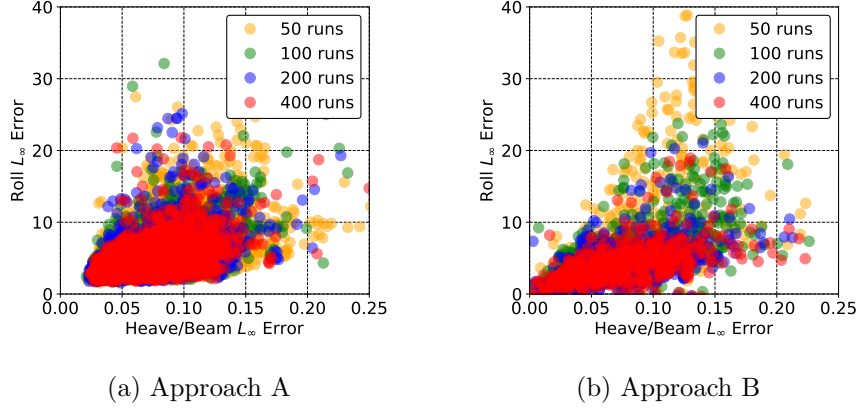


Figure 8: Comparison of the  $L_\infty$  error for each composite wave run with different LSTM models and varying amounts of training data.

Fig. 7 and 8 show comparisons of the CFD simulations and different LSTM models in a general sense that evaluates the similarity between the produced time-histories. However, the main criteria utilized in the CWG method is the absolute maximum response, due to each composite wave train that is employed to find critical wave groups that correspond to a specific threshold. The comparisons of absolute maximum roll in Fig. 9 show that for each composite wave train (depicted by a circle), the B models provide a more accurate depiction of the maximum roll response, while the A approach under-predicts the absolute maximum roll response on average. Similar to the comparison of the  $L_2$  and  $L_\infty$  error, the A approach does not seem to be influenced by the quantity of training data, while increasing the quantity improves the B models.

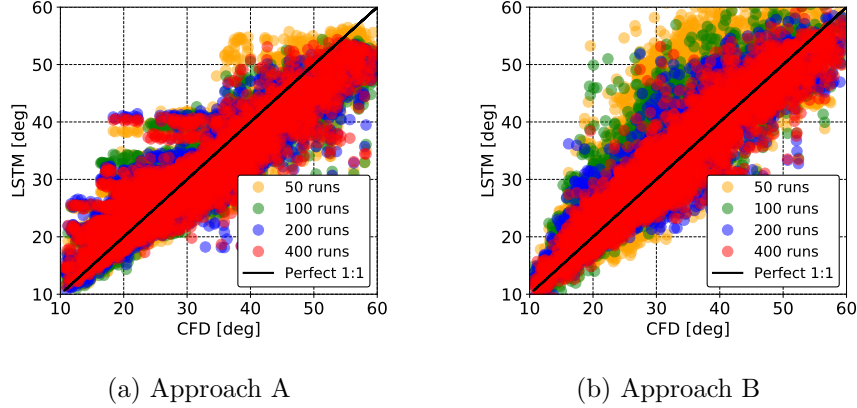


Figure 9: Comparison of the absolute maximum roll for each composite wave run with CFD and LSTM models with varying amounts of training data.

Fig. 10 further investigates the role of training data quantity and modeling approaches by comparing the tail of the probability distribution function (PDF) of the absolute value of roll response resulting from all the composite wave trains. The solid lines in Fig. 10 represent the kernel density estimates of the PDF, while the shaded regions labeled  $U_{CFD}$  and  $U_{LSTM}$  represent uncertainty estimates with an interval of two standard deviations, utilizing the moving block bootstrap (MBB) procedure discussed in [18]. The B approach demonstrates greater accuracy than the A approach, especially as the quantity of training data increases. The increase of training data has little to no effect on the convergence of the predictions for the A approach. However, the convergence of the B models is clear, and a comparison of the PDF tail like Fig. 10 will be useful in determining the convergence of B models in future work.

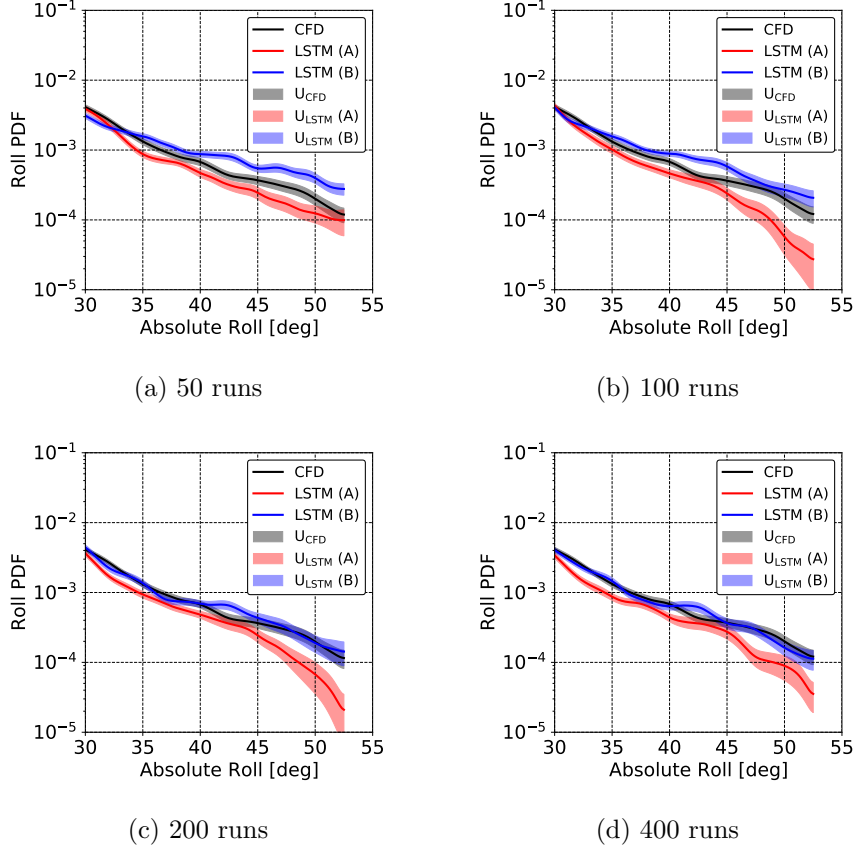


Figure 10: Comparison of the tail of the absolute roll PDF for CFD and LSTM models with varying amounts of training data.

The convergence of the models with training data is critical to the proper implementation of LSTM neural networks within the CWG-CFD framework for a reduction in total computational expense. Due to their lack of improvement with increasing training data quantity and diversity, the Approach A models would more than likely not benefit from any sampling schemes like those employed in [3, 4] to further reduce computational cost. Considering the current case

study, the B models can reproduce statistically representative extreme responses with 200-400 CFD simulations. An efficient sampling scheme of selecting the training runs based on model uncertainty could help reduce the computational cost and ensure the resulting framework is practical for everyday extreme ship response analyses.

## 5 CONCLUSIONS

LSTM neural networks are evaluated for their ability to predict extreme ship responses due to waves generated with the CWG-CFD method. Two different modeling approaches are validated against CFD simulations for a case study of a 2-D midship section of the ONRT in Sea State 7. The A modeling approach builds a single model for all composite wave trains, while the B approach develops multiple models that are trained with data from wave groups with the same parameters  $Tc$  and  $j$ . The B approach demonstrated a greater accuracy and convergence with respect to the quantity of training data. Approach A provided reasonable results but lower accuracy than the B approach and had little dependence on the quantity of training data. Therefore, the addition of more training data did not improve the quality of the A model predictions. The B approach is recommended for future work and demonstrates the effectiveness of the LSTM neural networks when incorporated into the CWG-CFD framework.

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